Standard Practice for
Use of Control Charts in Statistical Process Control[1]

1. Scope

1.1 This practice provides guidance for the use of control charts in statistical process control programs, which improve process quality through reducing variation by identifying and eliminating the effect of special causes of variation.

1.2 Control charts are used to continually monitor product or process characteristics to determine whether or not a process is in a state of statistical control. When this state is attained, the process characteristic will, at least approximately, vary within certain limits at a given probability.

1.3 This practice applies to variables data (characteristics measured on a continuous numerical scale) and to attributes data (characteristics measured as percentages, fractions, or counts of occurrences in a defined interval of time or space).

1.4 The system of units for this practice is not specified. Dimensional quantities in the practice are presented only as illustrations of calculation methods. The examples are not binding on products or test methods treated.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E456 Terminology Relating to Quality and Statistics

E1994 Practice for Use of Process Oriented AOQL and LTPD Sampling Plans

E2234 Practice for Sampling a Stream of Product by Attributes Indexed by AQL

E2281 Practice for Process Capability and Performance Measurement

E2762 Practice for Sampling a Stream of Product by Variables Indexed by AQL

3. Terminology

3.1 *Definitions:*

3.1.1 See Terminology E456 for a more extensive listing of statistical terms.

3.1.2 *assignable cause, n*—factor that contributes to variation in a process or product output that is feasible to detect and identify (see *special cause*).

3.1.2.1 *Discussion*—Many factors will contribute to variation, but it may not be feasible (economically or otherwise) to identify some of them.

3.1.3 *accepted reference value, ARV, n*—value that serves as an agreed-upon reference for comparison and is derived as: (*1*) a theoretical or established value based on scientific principles, (*2*) an assigned or certified value based on experimental work of some national or international organization, or (*3*) a consensus or certified value based on collaborative experimental work under the auspices of a scientific or engineering group. **E177**

3.1.4 *attributes data, n*—observed values or test results that indicate the presence or absence of specific characteristics or counts of occurrences of events in time or space.

3.1.5 *average run length (ARL), n*—the average number of times that a process will have been sampled and evaluated before a shift in process level is signaled.

3.1.5.1 *Discussion*—A long ARL is desirable for a process located at its specified level (so as to minimize calling for unneeded investigation or corrective action) and a short ARL is desirable for a process shifted to some undesirable level (so that corrective action will be called for promptly). ARL curves are used to describe the relative quickness in detecting level shifts of various control chart systems (see 5.1.4). The average number of units that will have been produced before a shift in level is signaled may also be of interest from an economic standpoint.

3.1.6 *c chart, n*—control chart that monitors the count of occurrences of an event in a defined increment of time or space.

3.1.7 *center line, n*—line on a control chart depicting the average level of the statistic being monitored.

3.1.8 *chance cause, n*—source of inherent random variation in a process which is predictable within statistical limits (see *common cause*).

3.1.8.1 *Discussion*—Chance causes may be unidentifiable, or may have known origins that are not easily controllable or cost effective to eliminate.

3.1.9 *common cause, n*—(see *chance cause*).

3.1.10 *control chart, n*—chart on which are plotted a statistical measure of a subgroup versus time of sampling along with limits based on the statistical distribution of that measure so as to indicate how much common, or chance, cause variation is inherent in the process or product.

3.1.11 *control chart factor, n*—a tabulated constant, depending on sample size, used to convert specified statistics or parameters into a central line value or control limit appropriate to the control chart.

3.1.12 *control limits, n*—limits on a control chart that are used as criteria for signaling the need for action or judging whether a set of data does or does not indicate a state of statistical control based on a prescribed degree of risk.

3.1.12.1 *Discussion*—For example, typical three-sigma limits carry a risk of 0.135 % of being out of control (on one side of the center line) when the process is actually in control and the statistic has a normal distribution.

3.1.13 *EWMA chart, n*—control chart that monitors the exponentially weighted moving averages of consecutive subgroups.

3.1.14 *EWMV chart, n*—control chart that monitors the exponentially weighted moving variance.

3.1.15 *exponentially weighted moving average (EWMA), n*—weighted average of time ordered data where the weights of past observations decrease geometrically with age.

3.1.15.1 *Discussion*—Data used for the EWMA may consist of individual observations, averages, fractions, numbers defective, or counts.

3.1.16 *exponentially weighted moving variance (EWMV), n*—weighted average of squared deviations of observations from their current estimate of the process average for time ordered observations, where the weights of past squared deviations decrease geometrically with age.

3.1.16.1 *Discussion*—The estimate of the process average used for the current deviation comes from a coupled EWMA chart monitoring the same process characteristic. This estimate is the EWMA from the previous time period, which is the forecast of the process average for the current time period.

3.1.17 *I chart, n*—control chart that monitors the individual subgroup observations.

3.1.18 *lower control limit (LCL), n*—minimum value of the control chart statistic that indicates statistical control.

3.1.19 *MR chart, n*—control chart that monitors the moving range of consecutive individual subgroup observations.

3.1.20 *p chart, n*—control chart that monitors the fraction of occurrences of an event.

3.1.21 *R chart, n*—control chart that monitors the range of observations within a subgroup.

3.1.22 *rational subgroup, n*—subgroup chosen to minimize the variability within subgroups and maximize the variability between subgroups (see *subgroup*).

3.1.22.1 *Discussion*—Variation within the subgroup is assumed to be due only to common, or chance, cause variation, that is, the variation is believed to be homogeneous. If using a range or standard deviation chart, this chart should be in statistical control. This implies that any assignable, or special, cause variation will show up as differences between the subgroups on a corresponding $\bar{X}$ chart.

3.1.23 *s chart, n*—control chart that monitors the standard deviations of subgroup observations.

3.1.24 *special cause, n*—(see *assignable cause*).

3.1.25 *standardized chart, n*—control chart that monitors a standardized statistic.

3.1.25.1 *Discussion*—A standardized statistic is equal to the statistic minus its mean and divided by its standard error.

3.1.26 *state of statistical control, n*—process condition when only common causes are operating on the process.

3.1.26.1 *Discussion*—In the strict sense, a process being in a state of statistical control implies that successive values of the characteristic have the statistical character of a sequence of observations drawn independently from a common distribution.

3.1.27 *statistical process control (SPC), n*—set of techniques for improving the quality of process output by reducing variability through the use of one or more control charts and a corrective action strategy used to bring the process back into a state of statistical control.

3.1.28 *subgroup, n*—set of observations on outputs sampled from a process at a particular time.

3.1.29 *u chart, n*—control chart that monitors the count of occurrences of an event in variable intervals of time or space, or another continuum.

3.1.30 *upper control limit (UCL), n*—maximum value of the control chart statistic that indicates statistical control.

3.1.31 *variables data, n*—observations or test results defined on a continuous scale.

3.1.32 *warning limits, n*—limits on a control chart that are two standard errors below and above the centerline.

3.1.33 *X-bar chart, n*—control chart that monitors the average of observations within a subgroup.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *allowance value, K, n*—amount of process shift to be detected.

3.2.2 *allowance multiplier, k, n*—multiplier of standard deviation that defines the allowance value, K.

3.2.3 *average count* $(\bar{c})$, *n*—arithmetic average of subgroup counts.

3.2.4 *average moving range* $(\overline{MR})$, *n*—arithmetic average of subgroup moving ranges.

3.2.5 *average proportion* $(\bar{p})$, *n*—arithmetic average of subgroup proportions.

3.2.6 *average range* $(\bar{R})$, *n*—arithmetic average of subgroup ranges.

3.2.7 *average standard deviation* $(\bar{s})$, *n*—arithmetic average of subgroup sample standard deviations.

3.2.8 *cumulative sum, CUSUM, n*—cumulative sum of deviations from the target value for time-ordered data.

3.2.8.1 *Discussion*—Data used for the CUSUM may consist of individual observations, subgroup averages, fractions defective, numbers defective, or counts.

3.2.9 *CUSUM chart, n*—control chart that monitors the cumulative sum of consecutive subgroups.

3.2.10 *decision interval, H, n*—the distance between the center line and the control limits.

3.2.11 *decision interval multiplier, h, n*—multiplier of standard deviation that defines the decision interval, H.

3.2.12 *grand average* $(\bar{\bar{X}})$, *n*—average of subgroup averages.

3.2.13 *inspection interval, n*—a subgroup size for counts of events in a defined interval of time space or another continuum.

3.2.13.1 *Discussion*—Examples are 10 000 metres of wire inspected for insulation defects, 100 square feet of material surface inspected for blemishes, the number of minor injuries per month, or scratches on bearing race surfaces.

3.2.14 *moving range (MR), n*—absolute difference between two adjacent subgroup observations in an *I* chart.

3.2.15 *observation, n*—a single value of a process output for charting purposes.

3.2.15.1 *Discussion*—This term has a different meaning than the term defined in Terminology E456, which refers there to a component of a test result.

3.2.16 *overall proportion, n*—average subgroup proportion calculated by dividing the total number of events by the total number of objects inspected (see average proportion).

3.2.16.1 *Discussion*—This calculation may be used for fixed or variable sample sizes.

3.2.17 *process, n*—set of interrelated or interacting activities that convert input into outputs.

3.2.18 *process target value, T, n*—target value for the observed process mean.

3.2.19 *relative size of process shift, δ, n*—size of process shift to detect in standard deviation units.

3.2.20 *subgroup average* $(\bar{X}_i)$, *n*—average for the *i*th subgroup in an *X*-bar chart.

3.2.21 *subgroup count* (c_i), *n*—count for the *i*th subgroup in a *c* chart.

3.2.22 *subgroup EWMA* (Z_i), *n*—value of the EWMA for the *i*th subgroup in an EWMA chart.

3.2.23 *subgroup EWMV* (V_i), *n*—value of the EWMV for the *i*th subgroup in an EWMV chart.

3.2.24 *subgroup individual observation* $(\bar{X}_i)$, *n*—value of the single observation for the *i*th subgroup in an *I* chart.

3.2.25 *subgroup moving range* (MR_i), *n*—moving range for the *i*th subgroup in an *MR* chart.

3.2.25.1 *Discussion*—If there are *k* subgroups, there will be *k*-1 moving ranges.

3.2.26 *subgroup proportion* (p_i), *n*—proportion for the *i*th subgroup in a *p* chart.

3.2.27 *subgroup range* (R_i), *n*—range of the observations for the *i*th subgroup in an *R* chart.

3.2.28 *subgroup size* (n_i), *n*—the number of observations, objects inspected, or the inspection interval in the *i*th subgroup.

3.2.28.1 *Discussion*—For fixed sample sizes the symbol *n* is used.

3.2.29 *subgroup standard deviation* (s_i), *n*—sample standard deviation of the observations for the *i*th subgroup in an *s* chart.

3.3 *Symbols:*

A_2 = factor for converting the average range to three standard errors for the *X*-bar chart (Table 1)

A_3 = factor for converting the average standard deviation to three standard errors of the average for the *X*-bar chart (Table 1)

B_3, B_4 = factors for converting the average standard deviation to three-sigma limits for the *s* chart (Table 1)

B_5^*, B_6^* = factors for converting the initial estimate of the variance to three-sigma limits for the EWMV chart (Table 11)

C_0 = cumulative sum (CUSUM) at time zero (12.2.2)

c_4 = factor for converting the average standard deviation to an unbiased estimate of sigma (*see* σ) (Table 1)

TABLE 1 Control Chart Factors

	for *X*-Bar and *R* Charts				for *X*-Bar and *S* Charts			
n	A_2	D_3	D_4	d_2	A_3	B_3	B_4	c_4
2	1.880	0	3.267	1.128	2.659	0	3.267	0.7979
3	1.023	0	2.575	1.693	1.954	0	2.568	0.8862
4	0.729	0	2.282	2.059	1.628	0	2.266	0.9213
5	0.577	0	2.114	2.326	1.427	0	2.089	0.9400
6	0.483	0	2.004	2.534	1.287	0.030	1.970	0.9515
7	0.419	0.076	1.924	2.704	1.182	0.118	1.882	0.9594
8	0.373	0.136	1.864	2.847	1.099	0.185	1.815	0.9650
9	0.337	0.184	1.816	2.970	1.032	0.239	1.761	0.9693
10	0.308	0.223	1.777	3.078	0.975	0.284	1.716	0.9727

Note: for larger numbers of *n*, see Ref. (1).[A]

[A] The boldface numbers in parentheses refer to a list of references at the end of this standard.

c_i	=	counts of the observed occurrences of events in the ith subgroup (10.2.1)
C_i	=	cumulative sum (CUSUM) at time, i (12.1)
$\bar{c}$	=	average of the k subgroup counts (10.2.1)
$\bar{d_2}$	=	factor for converting the average range to an estimate of sigma (see σ) (Table 1)
D_3, D_4	=	factors for converting the average range to three-sigma limits for the R chart (Table 1)
D_i^2	=	the squared deviation of the observation at time i minus its forecast average (13.1)
h	=	decision interval multiplier for calculation of the decision interval, H (12.1.5)
H	=	decision interval for calculation of CUSUM control limits (12.1.5)
k	=	number of subgroups used in calculation of control limits (6.2.1)
k	=	allowance multiplier for calculation of K (12.1.5.1)
K	=	amount of process shift to detect with a CUSUM chart (12.1.5)
MR_i	=	absolute value of the difference of the observations in the $(i\text{-}1)$th and the ith subgroups in a MR chart (8.2.1)
$\left(\overline{MR}\right)$	=	average of the subgroup moving ranges (8.2.2.1)
n	=	subgroup size, number of observations in a subgroup (5.1.3)
n_i	=	subgroup size, number of observations (objects inspected) in the ith subgroup (9.1.2)
p_i	=	proportion of the observed occurrences of events in the ith subgroup (9.2.1)
$\bar{p}$	=	average of the k subgroup proportions (9.2.1)
R_i	=	range of the observations in the ith subgroup for the R chart (6.2.1.2)
$\bar{R}$	=	average of the k subgroup ranges (6.2.2)
s_i	=	Sample standard deviation of the observations in the ith subgroup for the s chart (7.2.1)
s_z	=	standard error of the EWMA statistic (11.2.1.2)
$\bar{s}$	=	average of the k subgroup standard deviations (7.2.2)
T	=	process target value for process mean (12.1.1)
u_i	=	counts of the observed occurrences of events in the inspection interval divided by the size of the inspection interval for the ith subgroup (10.4.2)
V_0	=	exponentially-weighted moving variance at time zero (13.2.1)
V_i	=	exponentially-weighted moving variance statistic at time i (13.1)
X_i	=	single observation in the ith subgroup for the I chart (8.2.1)
X_{ij}	=	the jth observation in the ith subgroup for the X-bar chart (6.2.1)
$\bar{X}$	=	average of the individual observations over k subgroups for the I chart (8.2.2)
$\bar{X_i}$	=	average of the ith subgroup observations for the X-bar chart (6.2.1)
$\bar{\bar{X}}$	=	average of the k subgroup averages for the X-bar chart (6.2.2)
Y_i	=	value of the statistic being monitored by an EWMA chart at time i (11.2.1)
z_i	=	the standardized statistic for the ith subgroup (9.4.1.3)

Z_0	=	exponentially-weighted moving average at time zero (11.2.1.1)
Z_i	=	exponentially-weighted average (EWMA) statistic at time i (11.2.1)
δ	=	relative process shift for calculation of the allowance multiplier, k (12.1.5.1)
λ	=	factor $(0 < λ < 1)$ which determines the weighing of data in the EWMA statistic (11.2.1)
$\hat{σ}$	=	estimated common cause standard deviation of the process (6.2.4)
$\hat{σ_c}$	=	standard error of c, the number of observed counts (10.2.1.2)
$\hat{σ_p}$	=	standard error of p, the proportion of observed occurrences (9.2.2.4)
ν	=	effective degrees of freedom for the EWMV (13.1.2)
ω	=	factor $(0 < ω < 1)$ which determines the weighting of squared deviations in the EWMV statistic (13.1)

4. Significance and Use

4.1 This practice describes the use of control charts as a tool for use in statistical process control (SPC). Control charts were developed by Shewhart (2)[3] in the 1920s and are still in wide use today. SPC is a branch of statistical quality control (3, 4), which also encompasses process capability analysis and acceptance sampling inspection. Process capability analysis, as described in Practice E2281, requires the use of SPC in some of its procedures. Acceptance sampling inspection, described in Practices E1994, E2234, and E2762, requires the use of SPC so as to minimize rejection of product.

4.2 *Principles of SPC*—A process may be defined as a set of interrelated activities that convert inputs into outputs. SPC uses various statistical methodologies to improve the quality of a process by reducing the variability of one or more of its outputs, for example, a quality characteristic of a product or service.

4.2.1 A certain amount of variability will exist in all process outputs regardless of how well the process is designed or maintained. A process operating with only this inherent variability is said to be in a state of statistical control, with its output variability subject only to chance, or common, causes.

4.2.2 Process upsets, said to be due to assignable, or special causes, are manifested by changes in the output level, such as a spike, shift, trend, or by changes in the variability of an output. The control chart is the basic analytical tool in SPC and is used to detect the occurrence of special causes operating on the process.

4.2.3 When the control chart signals the presence of a special cause, other SPC tools, such as flow charts, brainstorming, cause-and-effect diagrams, or Pareto analysis, described in various references (4-8), are used to identify the special cause. Special causes, when identified, are either eliminated or controlled. When special cause variation is eliminated, process variability is reduced to its inherent variability, and control charts then function as a process

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[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.

monitor. Further reduction in variation would require modification of the process itself.

4.3 The use of control charts to adjust one or more process inputs is not recommended, although a control chart may signal the need to do so. Process adjustment schemes are outside the scope of this practice and are discussed by Box and Luceño (9).

4.4 The role of a control chart changes as the SPC program evolves. An SPC program can be organized into three stages (10).

4.4.1 *Stage A, Process Evaluation*—Historical data from the process are plotted on control charts to assess the current state of the process, and control limits from this data are calculated for further use. See Ref. (1) for a more complete discussion on the use of control charts for data analysis. Ideally, it is recommended that 100 or more numeric data points be collected for this stage. For single observations per subgroup at least 30 data points should be collected (6, 7). For attributes, a total of 20 to 25 subgroups of data are recommended. At this stage, it will be difficult to find special causes, but it would be useful to compile a list of possible sources for these for use in the next stage.

4.4.2 *Stage B, Process Improvement*—Process data are collected in real time and control charts, using limits calculated in Stage A, are used to detect special causes for identification and resolution. A team approach is vital for finding the sources of special cause variation, and process understanding will be increased. This stage is completed when further use of the control chart indicates that a state of statistical control exists.

4.4.3 *Stage C, Process Monitoring*—The control chart is used to monitor the process to confirm continually the state of statistical control and to react to new special causes entering the system or the reoccurrence of previous special causes. In the latter case, an out-of-control action plan (OCAP) can be developed to deal with this situation (7, 11). Update the control limits periodically or if process changes have occurred.

NOTE 1—Some practitioners combine Stages A and B into a Phase I and denote Stage C as Phase II (10).

5. Control Chart Principles and Usage

5.1 One or more observations of an output characteristic are periodically sampled from a process at a defined frequency. A control chart is basically a time plot summarizing these observations using a sample statistic, which is a function of the observations. The observations sampled at a particular time point constitute a subgroup. Control limits are plotted on the chart based on the sampling distribution of the sample statistic being evaluated (see 5.2 for further discussion).

NOTE 2—Subgroup statistics commonly used are the average, range, standard deviation, variance, percentage or fraction of an occurrence of an event among multiple opportunities, or the number of occurrences during a defined time period or in a defined space.

5.1.1 The subgroup sampling frequency is determined by practical considerations, such as time and cost of an observation, the process dynamics (how quickly the output responds to upsets), and consequences of not reacting promptly to a process upset.

NOTE 3—Sampling at too high of a frequency may introduce correlation between successive subgroups. This is referred to as autocorrelation. Control charts that can handle this type of correlation are outside the scope of this practice.

NOTE 4—Rules for nonrandomness (see 5.2.2) assume that the plotted points on the chart are independent of one another. This shall be kept in mind when determining the sampling frequency for the control charts discussed in this practice.

5.1.2 The sampling plan for collecting subgroup observations should be designed to minimize the variation of observations within a subgroup and to maximize variation between subgroups. This is termed *rational subgrouping*. This gives the best chance for the within-subgroup variation to estimate only the inherent, or common-cause, process variation.

NOTE 5—For example, to obtain hourly rational subgroups of size four in a product-filling operation, four bottles should be sampled within a short time span, rather than sampling one bottle every 15 min. Sampling over 1 h allows the admission of special cause variation as a component of within-subgroup variation.

5.1.3 The subgroup size, n, is the number of observations per subgroup. For ease of interpretation of the control chart, the subgroup size should be fixed (symbol n), and this is the usual case for variables data (see 5.3.1). In some situations, often involving retrospective data, variable subgroup sizes may be unavoidable, which is often the situation for attributes data (see 5.3.2).

5.1.4 *Subgroup Size and Average Run Length*—The average run length (ARL) is a measure of how quickly the control chart signals a sustained process shift of a given magnitude in the output characteristic being monitored. It is defined as the average number of subgroups needed to respond to a process shift of h sigma units, where sigma is the intrinsic standard deviation estimated by σ (see 6.2.4). The theoretical background for this relationship is developed in Montgomery (4), and Fig. 1 gives the curves relating ARL to the process shift for selected subgroup sizes in an *X*-bar chart. An ARL = 1 means that the next subgroup will have a very high probability of detecting the shift.

5.2 The control chart is a plot of the subgroup statistic in time order. The chart also features a center line, representing the time-averaged value of the statistic, and the lower and upper control limits, that are located at ±three standard errors of the statistic around the center line. The center line and control limits are calculated from the process data and are not based in any way on specification limits. The presence of a special cause is indicated by a subgroup statistic falling outside the control limits.

5.2.1 The use of three standard errors for control limits (so-called "three-sigma limits") was chosen by Shewhart (2), and therefore are also known as Shewhart Limits. Shewhart chose these limits to balance the two risks of: (*1*) failing to signal the presence of a special cause when one occurs, and (*2*) occurrence of an out-of-control signal when the process is actually in a state of statistical control (a false alarm).

5.2.2 Special cause variation may also be indicated by certain nonrandom patterns of the plotted subgroup statistic, as detected by using the so-called Western Electric Rules (3). To implement these rules, additional limits are shown on the chart at ±two standard errors (warning limits) and at ±one standard error (see 7.3 for example).
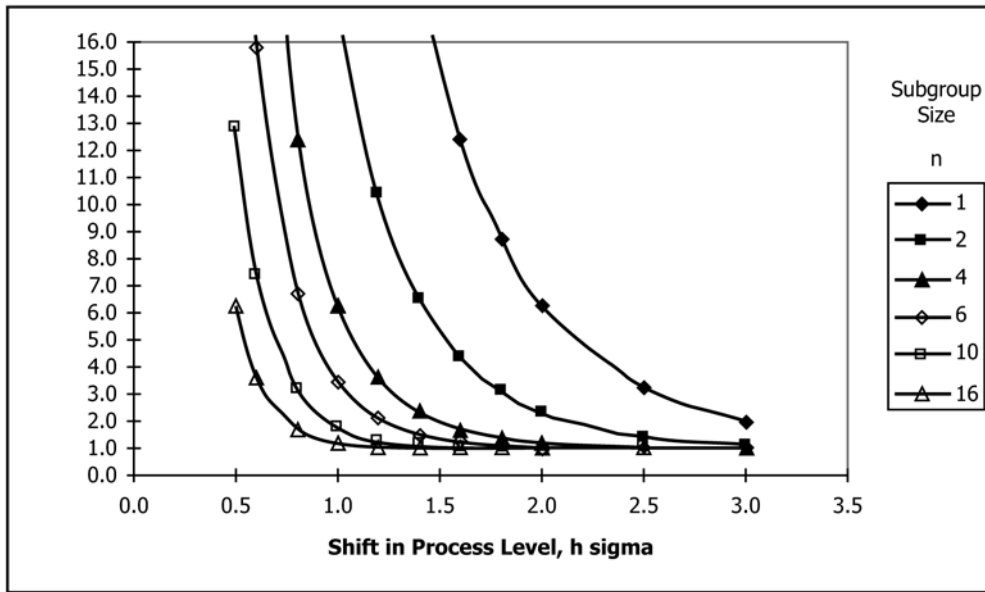
FIG. 1 ARL for the *X*-Bar Chart to Detect an *h*-Sigma Process Shift by Subgroup Size, *n*

5.2.2.1 *Western Electric Rules*—A shift in the process level is indicated if:

(1) One value falls outside either control limit,

(2) Two out of three consecutive values fall outside the warning limits on the same side,

(3) Four out of five consecutive values fall outside the ±one-sigma limits on the same side, and

(4) Eight consecutive values either fall above or fall below the center line.

5.2.2.2 Other Western Electric rules indicate less common situations of nonrandom behavior:

(1) Six consecutive values in a row are steadily increasing or decreasing (trend),

(2) Fifteen consecutive values are all within the ±one-sigma limits on either side of the center line,

(3) Fourteen consecutive values are alternating up and down, and

(4) Eight consecutive values are outside the ±one-sigma limits.

5.2.2.3 These rules should be used judiciously since they will increase the risk of a false alarm, in which the control chart indicates lack of statistical control when only common causes are operating. The effect of using each of the rules, and groups of these rules, on false alarm incidence is discussed by Champ and Woodall (**12**).

5.3 This practice describes the use of control charts for variables and attributes data.

5.3.1 Variables data represent observations obtained by observing and recording the magnitude of an output characteristic measured on a continuous numerical scale. Control charts are described for monitoring process variability and process level, and these two types of charts are used as a unit for process monitoring.

5.3.1.1 For multiple observations per subgroup, the subgroup average is the statistic for monitoring process level (*X*-bar chart) and either the subgroup range (*R* chart), or the subgroup standard deviation (*s* chart) is used for monitoring process variability. The range is easier to calculate and is nearly as efficient as the standard deviation for small subgroup sizes. The *X*-bar, *R* chart combination is discussed in Section 6. The *X*-bar, *s* chart combination is discussed in Section 7.

NOTE 6—For processes producing discrete items, a subgroup usually consists of multiple observations. The subgroup size is often five or less, but larger subgroup sizes may be used if measurement ease and cost are low. The larger the subgroup size, the more sensitive the control chart is to smaller shifts in the process level (see 5.1.4).

5.3.1.2 For single observations per subgroup, the subgroup individual observation is the statistic for monitoring process level (*I* chart) and the subgroup moving range is used for monitoring process variability (*MR* chart). The *I*, *MR* chart combination is discussed in Section 8.

NOTE 7—For batch or continuous processes producing bulk material, often only a single observation is taken per subgroup, as multiple observations would only reflect measurement variation.

5.3.2 Attributes data consist of two types: *(1)* observations representing the frequency of occurrence of an event in the subgroup, for example, the number or percentage of defective units in a subgroup of inspected units, or *(2)* observations representing the count of occurrences of an event in a defined interval of time or unit of space, for example, numbers of auto accidents per month in a given region. For attributes data, the standard error of the mean is a function of the process average, so that only a single control chart is needed.

NOTE 8—The subgroup size for attributes data, because of their lower cost and quicker measurement, is usually much greater than for numeric observations. Another reason is that variables data contain more information than attributes data, thus requiring a smaller subgroup size.

5.3.2.1 For monitoring the frequency of occurrences of an event with fixed subgroup size, the statistic is the proportion or fraction of objects having the attribute (*p* chart). An alternate statistic is the number of occurrences for a given subgroup size

(*np* chart) and these charts are described in Section 9. For monitoring with variable subgroup sizes, a modified *p* chart with variable control limits or a standardized control chart is used, and these charts are also described in Section 9.

5.3.2.2 For monitoring the count of occurrences over a defined time or space interval, termed the *inspection interval*, the statistic depends on whether or not the inspection interval is fixed or variable over subgroups. For a fixed inspection interval for all subgroups the statistic is the count (*c* chart); for variable inspection units the statistic is the count per inspection interval (*u* chart). Both charts are described in Section 10.

5.3.3 The EWMA chart plots the exponentially weighted moving average statistic which is described by Hunter (**13**). The EWMA may be calculated for individual observations and averages of multiple observations of variables data, and for percent defective, or counts of occurrences over time or space for attributes data. The calculations for the EWMA chart are defined and discussed in Section 11.

5.3.3.1 The EWMA chart is also a useful supplementary control chart to the previously discussed charts in SPC, and is a particularly good companion chart to the *I* chart for individual observations. The EWMA reacts more quickly to smaller shifts in the process characteristic, on the order of 1.5 standard errors or less, whereas the Shewhart-based charts are more sensitive to larger shifts. Examples of the EWMA chart as a supplementary chart are given in 11.4 and Appendix X1.

5.3.3.2 The EMWA chart is also used in process adjustment schemes where the EWMA statistic is used to locate the local mean of a non-stationary process and as a forecast of the next observation from the process. This usage is beyond the scope of this practice but is discussed by Box and Paniagua-Quiñones (**14**) and by Lucas and Saccucci (**15**).

5.3.4 The CUSUM chart plots the accumulated total value of differences between the measured values or monitored statistics and the predefined target or reference value as described in Montgomery (**4**). The CUSUM may be calculated for variables data using individual observations or subgroup averages and attributes data using percent defectives or counts of occurrences over time or space. The calculations for the CUSUM chart are defined and discussed in Section 12.

5.3.4.1 The CUSUM chart is used when smaller process shifts (1 to 1.5 sigma) are of interest. The CUSUM chart effectively detects a sustained small shift in the process mean or a slow process drift or trend. The CUSUM chart can also be used to evaluate the direction and the magnitude of the drift from the process target or reference value.

5.3.5 The CUSUM chart is not very effective in detecting large process shifts. Therefore, it is often used as a supplementary chart to I-chart or *X*-bar chart. In this case, either the I-chart or *X*-bar chart detects larger process shifts. The CUSUM chart detects smaller shifts (1 to 1.5 sigma) in process.

5.4 The EWMV chart is useful for monitoring the variance of a process characteristic from a continuous process where single measurements have been taken at each time point (see 5.3.1.2), and the EWMV chart may be considered as an alternative or companion to the Moving Range chart. The EWMV chart is based on the squared deviation of the current process observation from an estimate of the current process

average, which is obtained from a companion EWMA chart. The calculations for the EWMV chart are defined and discussed in Section 13.

6. Control Charts for Multiple Numerical Measurements per Subgroup (*X*-Bar, *R* Charts)

6.1 *Control Chart Usage*—These control charts are used for subgroups consisting of multiple numerical measurements. The *X*-bar chart is used for monitoring the process level, and the *R* chart is used for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

6.2 *Control Chart Setup and Calculations:*

6.2.1 Denote an observation X_{ij}, as the *j*th observation, $j = 1$, …, n, in the *i*th subgroup $i = 1$, …, k. For each of the k subgroups, calculate the *i*th subgroup average,

$$\bar{X}_i = \sum_{j=1}^{n} X_{ij}/n = (X_{i1} + X_{i2} \ldots + X_{in})/n \tag{1}$$

6.2.1.1 Averages may be rounded to one more significant figure than the data.

6.2.1.2 For each of the k subgroups, calculate the *i*th subgroup range, the difference between the largest and the smallest observation in the subgroup.

$$R_i = Max(X_{i1}, \ldots, X_{in}) - Min(X_{i1}, \ldots, X_{in}) \tag{2}$$

6.2.1.3 The averages and ranges are plotted as dots on the *X*-bar chart and the *R* chart, respectively. The dots may be connected by lines, if desired.

6.2.2 Calculate the grand average and the average range over all k subgroups:

$$\bar{\bar{X}} = \sum_{i=1}^{k} \bar{X}_i/k = (\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_k)/k \tag{3}$$

$$\bar{R} = \sum_{i=1}^{k} R_i/k = (R_1 + R_2 + \ldots + R_k)/k \tag{4}$$

6.2.2.1 These values are used for the center lines on the control chart, which are usually depicted as solid lines on the control chart, and may be rounded to one more significant figure than the data.

6.2.3 Using the control chart factors in Table 1, calculate the lower control limits (*LCL*) and upper control limits (*UCL*) for the two charts.

6.2.3.1 *For the X-Bar Chart:*

$$LCL = \bar{\bar{X}} - A_2\bar{R} \tag{5}$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} \tag{6}$$

6.2.3.2 *For the R Chart:*

$$LCL = D_3\bar{R} \tag{7}$$

$$UCL = D_4\bar{R} \tag{8}$$

6.2.3.3 The control limits are usually depicted as dashed lines on the control chart.

6.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{R}/d_2 \tag{9}$$

6.2.4.1 This estimate is useful in process capability studies (see Practice E2281).

6.2.5 Subgroup statistics falling outside the control limits on the X-bar chart or the R chart indicate the presence of a special cause. The Western Electric Rules may also be applied to the X-bar and R chart (see 5.2.2).

6.3 *Example—Liquid Product Filling into Bottles*—At a frequency of 30 min, four consecutive bottles are pulled from the filling line and weighed. The observations, subgroup averages, and subgroup ranges are listed in Table 2, and the grand average and average range are calculated at the bottom of the table.

6.3.1 The control limits are calculated as follows:

6.3.1.1 *X-Bar Chart:*

$$LCL = 246.44 - (0.729)(5.92) = 242.12$$
$$UCL = 246.44 + (0.729)(5.92) = 250.76$$

6.3.1.2 *R Chart:*

$$LCL = (0)(5.92) = 0$$
$$UCL = (2.282)(5.92) = 13.51$$

6.3.1.3 *Estimate of inherent standard deviation:*

$$\sigma = 5.92/2.059 = 2.87$$

6.3.1.4 The control charts are shown in Fig. 2 and Fig. 3 Both charts indicate that the filling weights are in statistical control.

7. Control Charts for Multiple Numerical Measurements per Subgroup (*X-Bar, s* Charts)

7.1 *Control Chart Usage*—These control charts are used for subgroups consisting of multiple numerical measurements, the X-bar chart for monitoring the process level, and the s chart for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

7.2 *Control Chart Setup and Calculations:*

7.2.1 Denote an observation X_{ij}, as the jth observation, $j = 1, \dots n$, in the ith subgroup, $i = 1, \dots, k$. For each of the k subgroups, calculate the ith subgroup average and the ith subgroup standard deviation:

$$\bar{X}_i = \sum_{j=1}^{n} X_{ij}/n = (X_{i1} + X_{i2} + \dots + X_{in})/n \tag{10}$$

$$s_i = \sqrt{\sum_{j=1}^{n} \left(X_{ij} - \bar{X}_i \right)^2/(n-1)} \tag{11}$$

7.2.1.1 Averages may be rounded to one more significant figure than the data.

7.2.1.2 Sample standard deviations may be rounded to two or three significant figures.

7.2.1.3 The averages and standard deviations are plotted as dots on the X-bar chart and the s chart, respectively. The dots may be connected by lines, if desired.

7.2.2 Calculate the grand average and the average standard deviation over all k subgroups:

$$\bar{\bar{X}} = \sum_{i=1}^{k} \bar{X}_i/k = \left(\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_k \right)/k \tag{12}$$

$$\bar{s} = \sum_{i=1}^{k} s_i/k = (s_1 + s_2 + \dots + s_k)/k \tag{13}$$

7.2.2.1 These values are used for the center lines on the control chart, usually depicted as solid lines, and may be rounded to the same number of significant figures as the subgroup statistics.

7.2.3 Using the control chart factors in Table 1, calculate the LCL and UCL for the two charts.

TABLE 2 Example of *X*-Bar, *R* Chart for Bottle-Filling Operation

| Subgroup	Bottle 1	Bottle 2	Bottle 3	Bottle 4	Average	Range
1	246.5	250.7	246.1	250.2	248.38	4.6
2	246.5	243.7	241.7	248.0	244.98	6.3
3	246.5	243.3	250.1	243.5	245.85	6.8
4	246.5	248.5	250.5	242.0	246.88	8.5
5	246.5	242.9	248.0	249.4	246.70	6.5
6	246.7	250.6	246.0	246.1	247.35	4.6
7	246.6	247.3	251.6	248.8	248.58	5.0
8	246.5	249.6	246.6	243.6	246.58	6.0
9	246.4	251.1	247.7	245.5	247.68	5.6
10	246.4	245.7	245.8	247.0	246.23	1.3
11	246.5	242.6	241.5	248.3	244.73	6.8
12	246.4	247.3	244.1	243.3	245.28	4.0
13	246.4	250.1	249.0	245.3	247.70	4.8
14	246.3	247.8	239.4	245.7	244.80	8.4
15	246.6	242.7	244.1	249.7	245.78	7.0
16	246.6	248.4	246.8	251.0	248.20	4.4
17	246.4	246.0	250.3	246.2	247.23	4.3
18	246.5	250.2	243.2	246.9	246.70	7.0
19	246.4	247.5	246.6	244.8	246.33	2.7
20	246.3	248.4	244.6	244.9	246.05	3.8
21	246.5	244.7	243.0	248.0	245.55	5.0
22	246.6	249.2	250.5	242.6	247.23	7.9
23	246.5	249.7	240.7	246.7	245.90	9.0
24	246.6	244.0	238.5	243.0	243.03	8.1
25	246.5	251.5	248.9	242.0	247.23	9.5
			Grand average	246.44		
			Average range			5.92
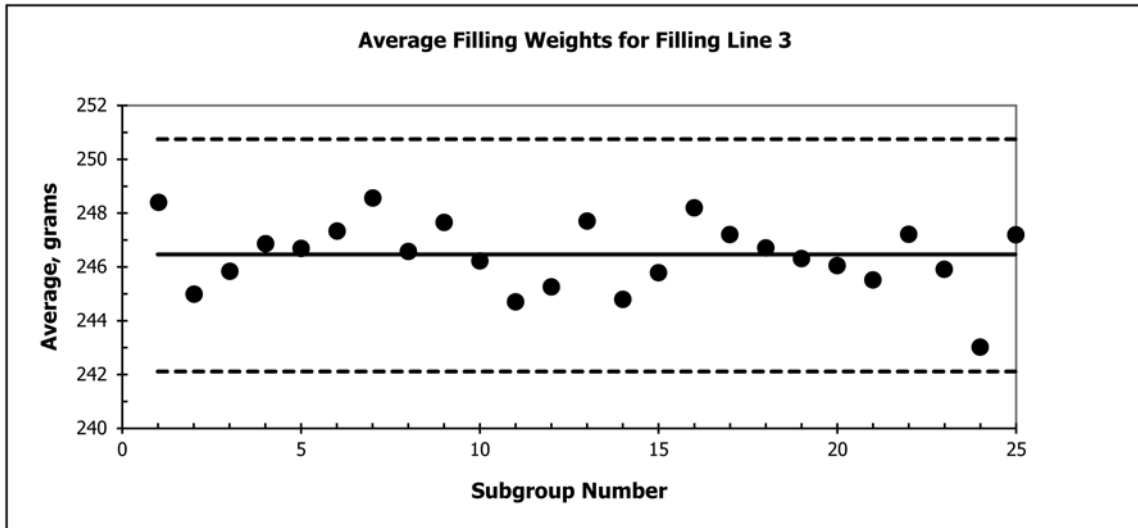
FIG. 2 X-Bar Chart for Filling Line 3
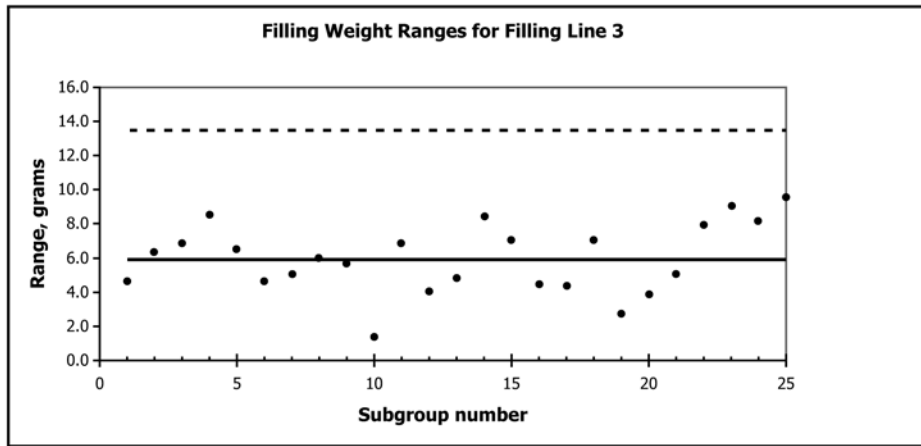
FIG. 3 Range Chart for Filling Line 3

7.2.3.1 *For the X-Bar Chart:*

$$LCL = \bar{\bar{X}} - A_3\bar{s} \tag{14}$$

$$UCL = \bar{\bar{X}} + A_3\bar{s} \tag{15}$$

7.2.3.2 *For the s Chart:*

$$LCL = B_3\bar{s} \tag{16}$$

$$UCL = B_4\bar{s} \tag{17}$$

7.2.3.3 The control limits are usually depicted by dashed lines on the control charts.

7.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{s}/c_4 \tag{18}$$

7.2.5 Subgroup statistics falling outside the control limits on the *X*-bar chart or the *s* chart indicate the presence of a special cause.

7.3 *Example*—Vitamin tablets are compressed from blended granulated powder and tablet hardness is measured on ten tablets each hour. The observations, subgroup averages, and subgroup standard deviations are listed in Table 3, and the grand average and average range are calculated at the bottom of the table.

7.3.1 The control limits are calculated as follows:

7.3.1.1 *X-Bar Chart:*

$$LCL = 24.141 - (0.975)(1.352) = 22.823$$

$$UCL = 24.141 + (0.975)(1.352) = 25.459$$

7.3.1.2 *The s Chart:*

$$LCL = (0.284)(1.352) = 0.384$$

$$UCL = (1.716)(1.352) = 2.320$$

7.3.2 The two-sigma warning limits and the one-sigma limits are also calculated for the *X*-bar chart to illustrate the use of the Western Electric Rules.

7.3.3 The warning limits and one-sigma limits for the *X*-bar chart were calculated as follows.

7.3.3.1 *Warning Limits:*

$$LCL = 24.141 - 2(0.975)(1.352)/3 = 23.262$$

$$UCL = 24.141 + 2(0.975)(1.352)/3 = 25.020$$

TABLE 3 Example of *X*-Bar, *S* Chart for Tablet Hardness

Subgroup	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	Avg	Std
1	21.3	19.5	21.3	23.1	22.4	24.6	23.4	22.4	21.4	22.9	22.23	1.419
2	21.4	22.2	22.1	23.3	23.9	22.9	21.6	24.6	25.7	24.1	23.18	1.399
3	23.9	24.2	22.8	22.9	25.9	21.4	23.1	20.5	23.6	23.8	23.21	1.494
4	23.4	26.3	24.4	25.3	22.0	25.8	22.7	26.5	21.6	25.0	24.30	1.781
5	25.6	22.9	24.6	23.8	23.6	22.7	27.1	25.3	25.1	25.5	24.62	1.372
6	26.8	25.7	25.6	24.8	25.7	26.0	23.6	22.8	22.1	24.7	24.78	1.507
7	26.6	24.0	25.0	26.3	25.3	25.2	25.0	24.1	22.6	26.0	25.01	1.199
8	26.7	25.9	22.6	23.8	27.3	26.7	25.7	24.1	25.2	25.2	25.32	1.470
9	25.0	24.8	22.4	22.9	24.9	24.4	22.5	22.7	23.8	24.0	23.74	1.037
10	26.1	25.3	25.7	25.0	24.7	26.2	23.6	24.2	25.1	24.3	25.02	0.844
							Center line				24.141	1.352
							LCL				22.823	0.384
							UCL				25.459	2.320
							Lower warning limit				23.262	
							Upper warning limit				25.020	
							Lower one-sigma limit				23.702	
							Upper one-sigma limit				24.580	

7.3.3.2 *One-Sigma Limits:*

$$LCL = 24.141 - (0.975)(1.352)/3 = 23.702$$

$$UCL = 24.141 + (0.975)(1.352)/3 = 24.580$$

7.3.3.3 *Estimate of Inherent Standard Deviation:*

$$\hat{\sigma} = 1.352/0.9727 = 1.39$$

7.3.3.4 The control charts are shown in Fig. 4 and Fig. 5. The *s* chart indicates statistical control in the process variation.

7.3.4 *The X-Bar Chart Gives Several Out-of-Control Signals:*

7.3.4.1 *Subgroup 1*—Below the *LCL*.

7.3.4.2 *Subgroups 2 and 3*—Two points outside the warning limit on the same side.

7.3.4.3 *Subgroups 6, 7, and 8*—End points of six points in a row steadily increasing.

7.3.4.4 *Subgroup 10*—Four out of five points on the same side of the upper one-sigma limits.

7.3.5 It appears that the process level has been steadily increasing during the run. Some possible special causes are particle segregation in the feed hopper or a drift in the press settings.

8. Control Charts for Single Numerical Measurements per Subgroup (*I*, *MR* Charts)

8.1 *Control Chart Usage*—These control charts are used for subgroups consisting of a single numerical measurement. The *I* chart is used for monitoring the process level and the *MR* chart is used for monitoring the short-term variability. The two charts are used as a unit for SPC purposes, although some practitioners state that the *MR* chart does not add value and recommend against its use for other than calculating the control limits for the *I* chart (**16**).

8.2 *Control Chart Setup and Calculations:*

8.2.1 Denote the observation, X_i, as the individual observation in the *i*th subgroup, $i = 1, 2,…, k$.

8.2.1.1 Note that the first subgroup will not have a moving range. For the $k–1$ subgroups, $i = 2, …, k$ calculate the moving range, the absolute value of the difference between two successive values:

$$MR_i = |X_i - X_{i-1}| \tag{19}$$
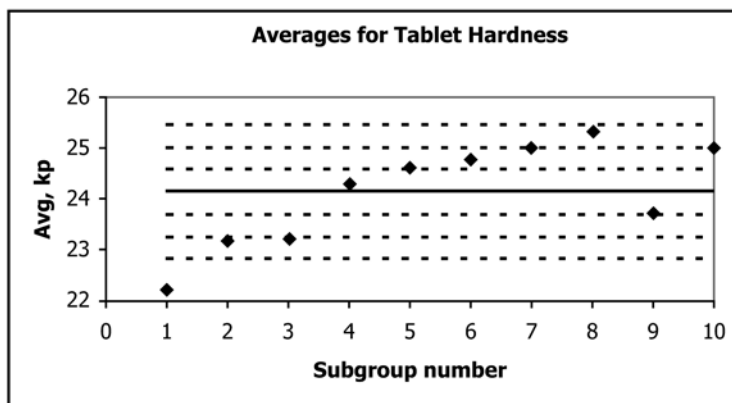
FIG. 4 *X*-Bar Chart for Tablet Hardness
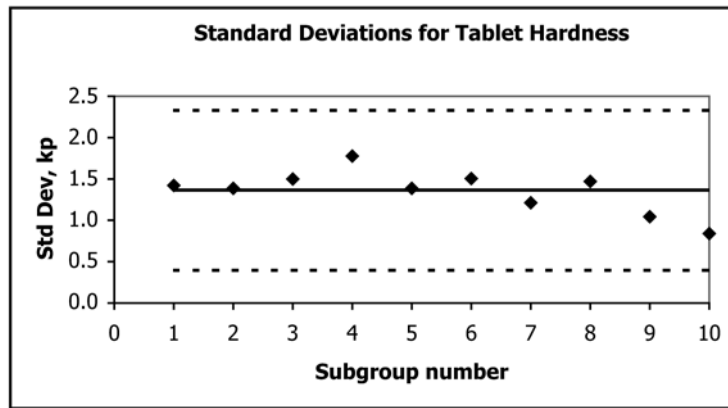
FIG. 5 s Chart for Tablet Hardness

8.2.1.2 The individual values and moving ranges are plotted as dots on the *I* chart and the *MR* chart, respectively. The dots may be connected by lines, if desired.

8.2.2 Calculate the average of the observations over all *k* subgroups:

$$\bar{X} = \sum_{i=1}^{k} X_i/k = (X_1 + X_2 + \ldots + X_k)/k \qquad (20)$$

Also calculate the average moving range for the *k*–1 subgroups:

$$\overline{MR} = \sum_{i=2}^{k} MR_i/(k-1) = (MR_2 + MR_3 + \ldots + MR_k)/(k-1) \qquad (21)$$

8.2.2.1 These values are used for the center lines on the control charts, usually depicted as a solid line, and may be rounded to one more significant figure than the data.

8.2.3 Calculate the control limits, *LCL* and *UCL*, for the two charts.

8.2.3.1 *For the I Chart:*

$$LCL = \bar{X} - 2.66\overline{MR} \qquad (22)$$

$$UCL = \bar{X} + 2.66\overline{MR} \qquad (23)$$

8.2.3.2 *For the MR Chart:*

$$LCL = 0 \qquad (24)$$

$$UCL = 3.27\overline{MR} \qquad (25)$$

8.2.3.3 The control limits are usually depicted as dashed lines on the charts.

8.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \overline{MR}/1.128 \qquad (26)$$

8.2.5 Subgroup statistics falling outside the control limits on the *I* chart or the *MR* chart indicate the presence of a special cause. The Western Electric Rules may also be applied to the *I* chart (see 5.2.1).

8.3 *Example*—Batches of polymer are sampled and analyzed for an impurity reported as weight percent. The values for 30 batches are listed in Table 4 along with the calculated moving ranges. The average impurity and average moving range are also listed along with the *LCL* and *UCL* for the charts.

TABLE 4 Example of I, MR Chart for % Impurity in Polymer Batches

Subgroup	Impurity	MR
1	1.39	
2	1.42	0.03
3	1.42	0.00
4	1.39	0.03
5	1.40	0.01
6	1.46	0.06
7	1.70	0.24
8	1.33	0.37
9	1.36	0.03
10	1.50	0.14
11	1.38	0.12
12	1.28	0.10
13	1.68	0.40
14	1.35	0.33
15	1.38	0.03
16	1.30	0.08
17	1.44	0.14
18	1.47	0.03
19	1.38	0.09
20	1.54	0.16
21	1.38	0.16
22	1.34	0.04
23	1.91	0.57
24	1.24	0.67
25	1.45	0.21
26	1.55	0.10
27	1.35	0.20
28	1.45	0.10
29	1.56	0.11
30	1.32	0.24
Center	1.437	0.165
LCL	0.998	0
UCL	1.877	0.540

8.3.1 The control limits are calculated as follows:

8.3.1.1 *I Chart:*

$$LCL = 1.437 - (2.66)(0.165) = 0.998$$

$$UCL = 1.437 + (2.66)(0.165) = 1.877$$

8.3.1.2 *MR Chart:*

$$LCL = (0)(0.165) = 0$$

$$UCL = (3.27)(0.165) = 0.540$$

8.3.2 The *I* chart indicates an out-of-control point at Subgroup 23 (Fig. 6). The *MR* chart indicates out-of-control points at Subgroups 23 and 24 (Fig. 7) and this was a result of the
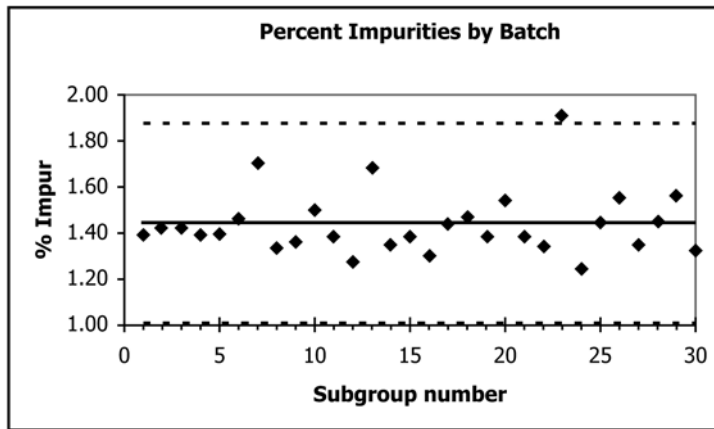
FIG. 6 *I* Chart for Batch Impurities
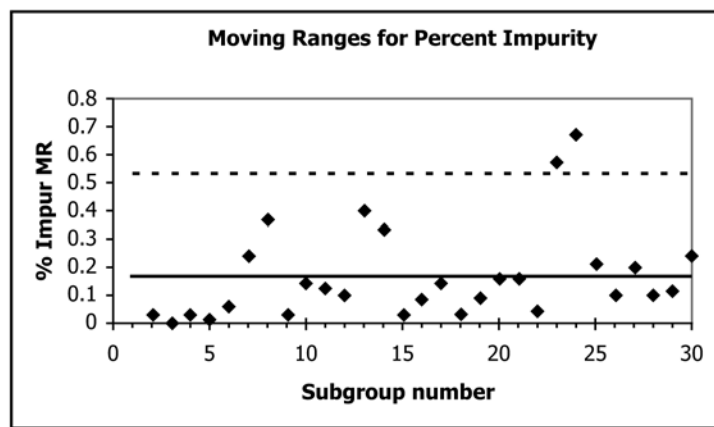
FIG. 7 *MR* Chart for % Impurities

high impurity value for Subgroup 23. This illustrates that the *MR* chart is affected by the successive differences between individual observations, and thus, it is more difficult to exclude special cause variation from entering the picture in the *MR* chart.

9. Control Charts for Fraction and Number of Occurrences (*p*, *np*, and Standardized Charts)

9.1 *Control Chart Usage*—These control charts are used for subgroups consisting of the fraction occurrence of an event, the *p* chart, or number of occurrences, the *np* chart. For example, the occurrence could be nonconformance of a manufactured unit with respect to a specification limit.

9.1.1 In routine monitoring use, the subgroup size is fixed (symbol *n*). Resulting *p* charts are easier to set up and interpret, since the control limits will be uniform for all subgroups.

9.1.2 In the retrospective analysis of data, the subgroup size may be variable (symbol n_i). This will result in differing sets of control limits that are dependent on the subgroup size.

9.2 *Control Chart Setup and Calculations for Fixed Subgroup Size:*

9.2.1 Denote an observation X_{ij}, as the *j*th observation in the *i*th subgroup, where $X_{ij} = 1$ if there was an occurrence of the attribute, for example, defect, and $X_{ij} = 0$ if there is no occurrence. Let X_i denote the number of occurrences for the *i*th subgroup

$$X_i = \sum_{j=1}^{n} X_{ij} \qquad (27)$$

9.2.1.1 Calculate the fraction occurrence p_i for the *i*th subgroup.

$$p_i = \sum_{j=1}^{n} X_{ij}/n = X_i/n \qquad (28)$$

9.2.1.2 The fractions are plotted as dots on the *p* chart. The numbers of occurrences are plotted as dots on the *np* chart. The dots may be connected by lines, if desired.

9.2.2 Calculate the average fraction occurrence over all *k* subgroups:

$$\bar{p} = \sum_{i=1}^{k} p_i/k = (p_1 + p_2 + ... + p_k)/k \qquad (29)$$

9.2.2.1 This value is used for the center line on the *p* chart.

9.2.2.2 The center line for the *np* chart is $n\bar{p}$.

9.2.2.3 Center lines are usually depicted as solid lines on the control chart.

9.2.2.4 *Calculate the Standard Error for p:*

$$\sigma_p = \sqrt{\bar{p}\,(1-\bar{p})/n} \qquad (30)$$

9.2.3 Calculate the control limits, *LCL* and *UCL*, for the two charts.

9.2.3.1 *For the p Chart:*

$$LCL = \bar{p} - 3\sigma_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} \qquad (31)$$

$$UCL = \bar{p} + 3\sigma_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} \qquad (32)$$

If the calculated *LCL* is negative, this limit is set to zero.

9.2.3.2 Control limits are usually depicted as dashed lines on the control charts.

9.2.4 The *np* chart center line is $n\bar{p}$ with control limits as follows:

9.2.4.1 *For the np Chart:*

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \qquad (33)$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \qquad (34)$$

If the calculated *LCL* is negative then this limit is set to zero.

NOTE 9—The *np* chart should only be used when the sample size for each subgroup is constant.

9.2.5 Subgroup statistics falling outside the control limits on the *p* chart or the *np* chart indicate the presence of a special cause.

9.3 *Example—*Cartons are inspected each shift in samples of 200 for minor (cosmetic) defects (such as tearing, dents, or scoring). Table 5 lists the number of nonconforming cartons (*X*) and the fraction defective (*p*) for 30 inspections. The *p* chart is shown in Fig. 8 and the *np* chart is shown in Fig. 9. The *np* chart is identical to the *p* chart but with the vertical scale multiplied by *n*. Special cause variation is indicated for Subgroups 15 and 23.

9.3.1 The control limits are calculated as follows:

9.3.1.1 *p Chart:*

$$LCL = 0.058 - 3\sqrt{(0.058)(1-0.058)/200} = 0.008$$

$$UCL = 0.058 + 3\sqrt{(0.058)(1-0.058)/200} = 0.107$$

9.3.1.2 *np Chart:*

$$LCL = 11.6 - 3\sqrt{(200)(0.058)(1-0.058)} = 1.7$$

$$UCL = 11.6 + 3\sqrt{(200)(0.058)(1-0.058)} = 21.5$$

TABLE 5 Example of *p* Chart for Nonconforming Cartons in Samples of 200

Sub	X	p	Sub	X	p
1	12	0.060	21	20	0.100
2	15	0.075	22	18	0.090
3	8	0.040	23	24	0.120
4	10	0.050	24	15	0.075
5	4	0.020	25	9	0.045
6	7	0.035	26	12	0.060
7	16	0.080	27	7	0.035
8	9	0.045	28	13	0.065
9	14	0.070	29	9	0.045
10	10	0.050	30	6	0.030
11	5	0.025			
12	6	0.030	Average	11.6	0.058
13	17	0.085	LCL	1.7	0.008
14	12	0.060	UCL	21.5	0.107
15	22	0.110		NP	P
16	8	0.040			
17	10	0.050			
18	5	0.025			
19	13	0.065			
20	11	0.055			

9.3.2 Except for a change in the vertical scale, the two control charts are identical.

Out-of-control signals are indicated at Subgroups 15 and 23.

9.4 *Control Chart Setup and Calculations for Variable Subgroup Size:*

9.4.1 There are three approaches to dealing with this situation.

9.4.1.1 Use an average subgroup size and use this for the calculations in 9.2. This is not recommended if the subgroup sizes differ widely in value, say greater than ±10 %.

$$\bar{n} = \sum_{i=1}^{k} n_i / k \qquad (35)$$

9.4.1.2 Use a chart with variable control limits. Calculate control limits for each subgroup based on the subgroup size. The center line is calculated from the overall proportion, the total number of objects having the attribute divided by the total number of objects inspected.

$$CL = \bar{p} = \sum_{i=1}^{k} x_i / \sum_{i=1}^{k} n_i \qquad (36)$$

The control limits for the *i*th subgroup are:

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \qquad (37)$$

9.4.1.3 Use a standardized chart. This method calculates a standardized proportion:

$$z_i = \frac{(p_i - \bar{p})}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}} \qquad (38)$$

which subtracts the process mean and divides by the process standard deviation for each subgroup. Each standardized value has an expected mean of zero with standard deviation of one. The center line is zero and the control limits are –3 and 3. This chart has a more uniform appearance than the chart with varying control limits.

9.5 *Example—*On a product information hot line, the proportion of daily calls involving product complaints were of interest. The numbers of daily calls and calls involving complaints are listed in Table 6. The proportion of calls involving complaints were calculated and the totals over 24 days are also listed in the table.

9.5.1 The control limits were calculated as follows:

9.5.1.1 *For the p Chart:*

CL = 233/863 = 0.27

The LCL and UCL values for each subgroup are listed in Table 6.

The *p* chart with variable limits is depicted in Fig. 10, and indicated an out-of-control proportion on Day 13.

9.5.1.2 *For the Standardized Chart:*

The individual standardized proportion values are listed in the last column of Table 6.

For the standardized chart CL = 0, LCL = –3, and UCL = 3.

The standardized chart is depicted in Fig. 11, and also indicated an out-of-control proportion on Day 13.
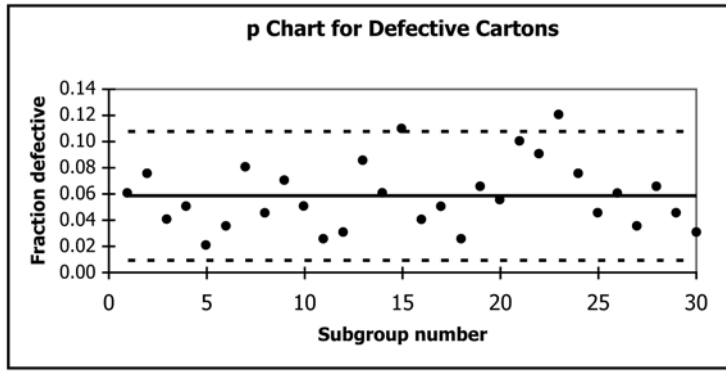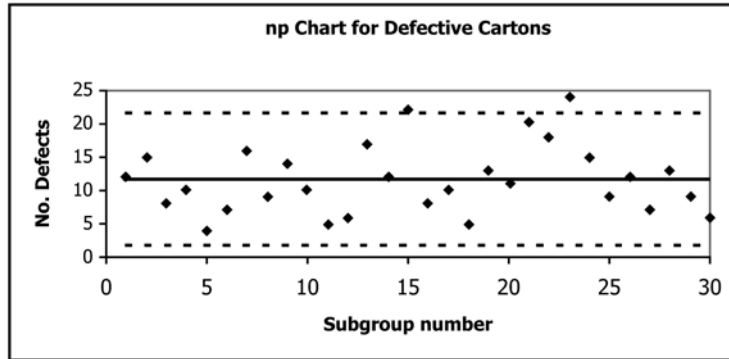
FIG. 8 *p* Chart for Nonconforming Cartons

FIG. 9 *np* Chart for Nonconforming Cartons

TABLE 6 Example of *p* for Variable Subgroup Sizes and Standardized Chart Applied to Proportion of Daily Calls Involving Complaints

Day	Calls	Complaints	Proportion	*p*-Chart Control Limits			Standardized *p*
	n_i	X_i	p	LCL	CL	UCL	z_i
1	25	5	0.20	0.004	0.270	0.536	−0.79
2	34	7	0.21	0.042	0.270	0.498	−0.84
3	56	14	0.25	0.092	0.270	0.448	−0.34
4	43	7	0.16	0.067	0.270	0.473	−1.58
5	36	13	0.36	0.048	0.270	0.492	1.23
6	42	17	0.40	0.064	0.270	0.475	1.97
7	21	3	0.14	0.000	0.270	0.561	−1.31
8	24	8	0.33	0.000	0.270	0.542	0.70
9	36	5	0.14	0.048	0.270	0.492	−1.77
10	29	9	0.31	0.023	0.270	0.517	0.49
11	41	15	0.37	0.062	0.270	0.478	1.38
12	35	12	0.34	0.045	0.270	0.495	0.97
13	34	18	0.53	0.042	0.270	0.498	3.41
14	37	6	0.16	0.051	0.270	0.489	−1.48
15	41	9	0.22	0.062	0.270	0.478	−0.73
16	42	17	0.40	0.064	0.270	0.475	1.97
17	43	5	0.12	0.067	0.270	0.473	−2.27
18	23	6	0.26	0.000	0.270	0.548	−0.10
19	38	16	0.42	0.054	0.270	0.486	2.10
20	47	14	0.30	0.076	0.270	0.464	0.43
21	30	7	0.23	0.027	0.270	0.513	−0.45
22	42	3	0.07	0.064	0.270	0.475	−2.90
23	33	14	0.42	0.038	0.270	0.502	2.00
24	31	3	0.10	0.031	0.270	0.509	−2.17
Total	863	233					

10. Control Charts for Counts of Occurrences in a Defined Time or Space Increment (*c* Chart)

10.1 *Control Chart Usage*—These control charts are used for subgroups consisting of the counts of occurrences of events over a defined time or space interval, within which there are multiple opportunities for occurrence of an event. For example, the event might be the occurrence of a knot of a specified
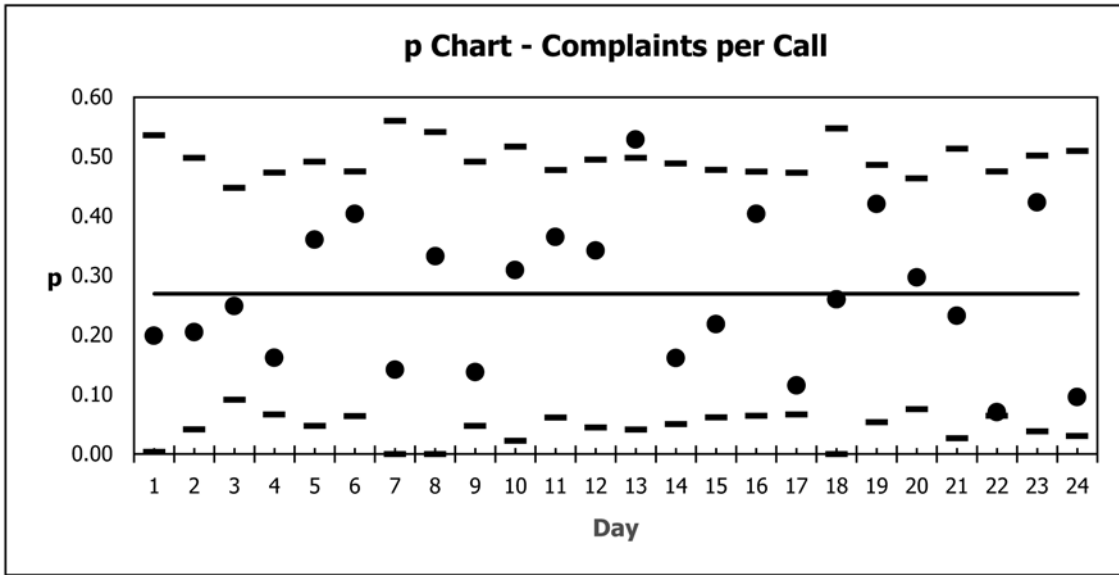
14

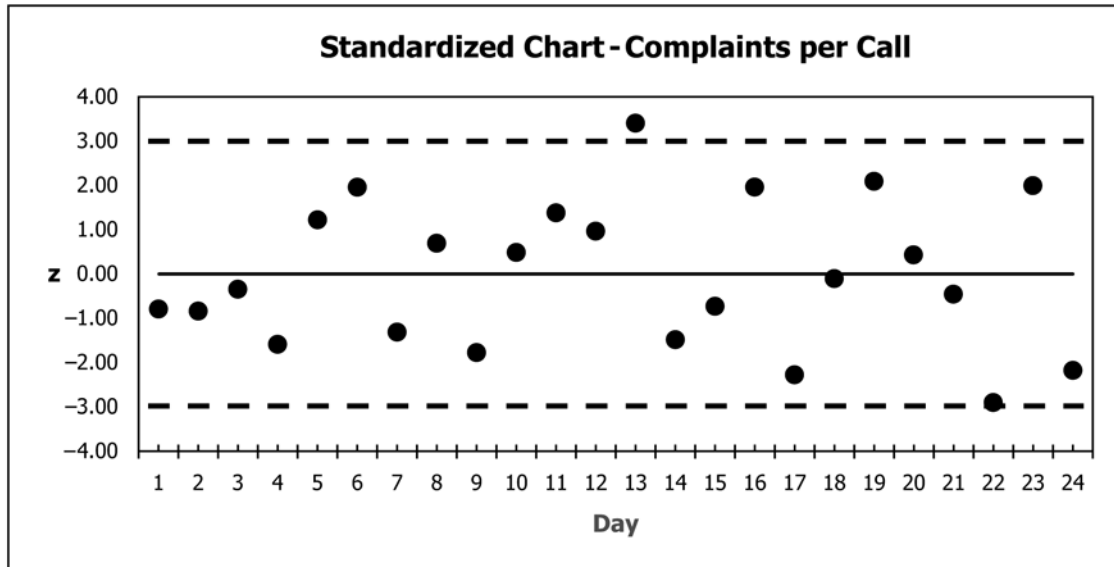FIG. 10 *p* Chart for Variable Subgroup Size Example

FIG. 11 Standardized Chart for Variable Subgroup Size Example

diameter on a wood panel or the number of minor injuries per 10 000 hours worked in a manufacturing plant.

10.1.1 The defined time or space interval, considered as a nominal subgroup size, may be fixed, as noted in 9.1.1. The *c* chart is used for these situations. The nominal subgroup size is termed an *inspection interval*.

10.1.2 The defined time or space interval may be variable, as noted in 9.1.2. When the subgroup size varies it may be defined as a fraction of the inspection interval. For example, if the inspection interval is defined as 100 square feet, then a subgroup size of 200 square feet would constitute two inspection units. A subgroup size of 80 square feet would be 0.8 inspection units. The *u* chart is used for these situations.

10.2 *Control Chart Setup and Calculations for c Chart:*

10.2.1 Denote the number of occurrences in the subgroup as c_i for the *i*th subgroup. Calculate the average count over all *k* subgroups:

$$\bar{c} = \sum_{i=1}^{k} c_i/k = (c_1 + c_2 + \ldots + c_k)/k \qquad (39)$$

10.2.1.1 This value is used for the center line on the *c* chart. Center lines are usually depicted as solid lines on the control chart.

10.2.1.2 Calculate the standard error of *c*:

$$\sigma_c = \sqrt{\bar{c}} \qquad (40)$$

10.2.2 Calculate the *LCL* and *UCL* for the chart.

10.2.2.1 *For the c Chart:*

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$$LCL = \bar{c} - 3\sigma_c = \bar{c} - 3\sqrt{\bar{c}} \qquad (41)$$

$$UCL = \bar{c} + 3\sigma_c = \bar{c} + 3\sqrt{\bar{c}} \qquad (42)$$

If the calculated *LCL* is negative then this limit is set to zero.

10.2.2.2 Control limits are usually depicted as dashed lines on the control charts.

10.2.3 Subgroup statistics falling outside the control limits on the *c* chart indicate the presence of a special cause.

10.3 *Example*—The number of minor injuries per month are tracked for a manufacturing plant with a stable workforce over a two-year period (Table 7). A *c* chart (Fig. 12) indicated that the injuries evolved from a common cause system. Although the eight minor injuries during Month 10 seemed unusually high, it is within the normal range of variation, and it might not be fruitful to investigate for a special cause.

10.3.1 The control limits are calculated as follows:

10.3.1.1 *c Chart:*

$$LCL = 3.3 - 3\sqrt{3.3} = -2.2 \text{ , so set limit to } 0$$

$$UCL = 3.3 + 3\sqrt{3.3} = 8.7$$

10.4 *Control Chart Setup and Calculations for u Chart:*

10.4.1 Denote the count of occurrences in the *i*th subgroup as c_i. Calculate the size of the inspection interval (symbol n_i) for the *i*th subgroup. Note that n_i does not have to be an integer value.

10.4.2 Calculate the number of occurrences per inspection unit for the *i*th subgroup.

$$u_i = c_i / n_i \qquad (43)$$

10.4.3 Calculate the average number of occurrences per inspection unit (symbol $\bar{u}$) over the *k* subgroups. Use this value for the centerline of the control chart.

$$\bar{u} = \sum_{i=1}^{k} c_i / \sum_{i=1}^{k} n_i \qquad (44)$$

Center lines are usually depicted as solid lines on the control chart.

10.4.4 Calculate the lower and upper control limits for each subgroup, usually depicted as dashed lines on the control chart. If the calculated LCL is negative then this limit is set to zero.

$$LCL_i = \bar{u} - 3\sqrt{\bar{u}/n_i} \qquad (45)$$

TABLE 7 *c* Chart Example

Month	c	Month	c
1	5	13	0
2	3	14	4
3	2	15	6
4	4	16	2
5	0	17	4
6	3	18	5
7	4	19	3
8	1	20	6
9	4	21	2
10	8	22	3
11	3	23	1
12	1	24	5
Average			3.3
LCL			–2.2
UCL			8.7

$$UCL_i = \bar{u} + 3\sqrt{\bar{u}/n_i} \qquad (46)$$

10.4.5 Subgroup statistics falling outside the control limits on the *u* chart indicate the presence of a special cause.

10.4.6 *Standardized Chart*—This method calculates a standardized value for each subgroup:

$$z_i = \frac{u_i - \bar{u}}{\sqrt{\bar{u}/n_i}} \qquad (47)$$

On the control chart center line is zero and the control limits are –3 and 3. This chart has a more uniform appearance than the chart with varying control limits.

10.5 *Example*—Units of fabric used for rugs are inspected for defects, with an inspection interval of 100 square yards. The fabric pieces are manufactured in three sizes of 100, 200, and 300 square feet. During production a piece is inspected for defects on a periodic basis.

10.5.1 Table 8 lists the areas inspected and the numbers of defects for each subgroup. The table lists the calculated *u* values for each subgroup.

10.5.2 The average *u* calculated as the ratio of total defects to the total inspection intervals shown on the bottom line of the table. For this example $\bar{u} = 90/60 = 1.5$. This value is the CL for the chart.

10.5.3 The UCL's were calculated for each subgroup and are listed in Table 8. All of the LCL values were negative so the LCL was zero for all subgroups.

10.5.4 The control chart is depicted in Fig. 13. For subgroup 5, *u* equals 5.0 versus the UCL of 5.2, which did not indicate an out-of-control value. All subgroups fell below their UCL's, indicating statistical control.

10.5.5 The standardized values are listed in the last column of Table 8. The standardized chart is depicted in Fig. 14. For subgroup 5, *z* equals 2.9 versus the UCL of 3.0, which did not indicate an out-of-control value. All subgroups fell within the control limits, indicating statistical control.

11. Control Chart Using the Exponentially-Weighted Moving Average (EWMA Chart)

11.1 *Control Chart Usage*—The EWMA control chart uses the exponentially-weighted moving average (EWMA) statistic, a type of weighted average of numerical subgroup statistics, where the weights decrease geometrically with the age of the subgroup. A weighting factor λ is used to control the rate of decrease of the weights with time.

11.1.1 Denote as Y_i the subgroup statistic at time *i*. This statistic may be an individual observation or an average, a percentage, or a count based on multiple observations. The EWMA statistic at time *i* is denoted Z_i and is calculated as:

$$Z_i = \lambda Y_i + (1 - \lambda) Z_{i-1} \qquad (48)$$

where Z_{i-1} is the EWMA at the immediately preceding time *i*-1 and λ is the weighting factor $0 < \lambda < 1$ for combining the current subgroup statistic Y_i and the preceding EWMA to obtain the current EWMA.

11.1.1.1 Typical values of λ used for EWMA charts range between 0.1 and 0.5, but the performance of the EWMA chart is often robust to the value chosen. A value for λ can be
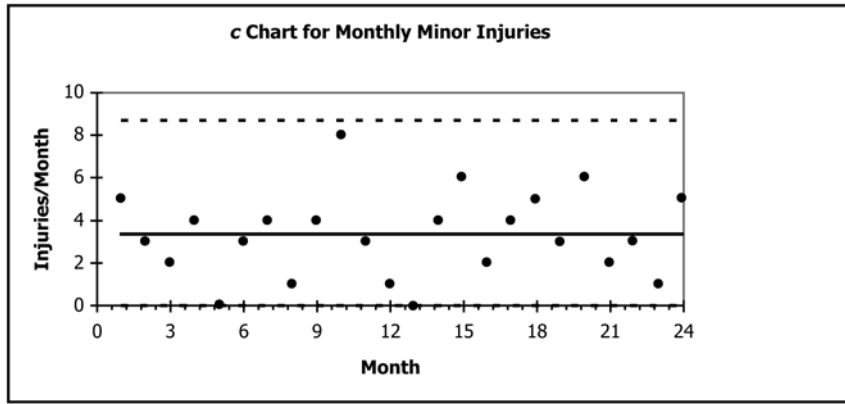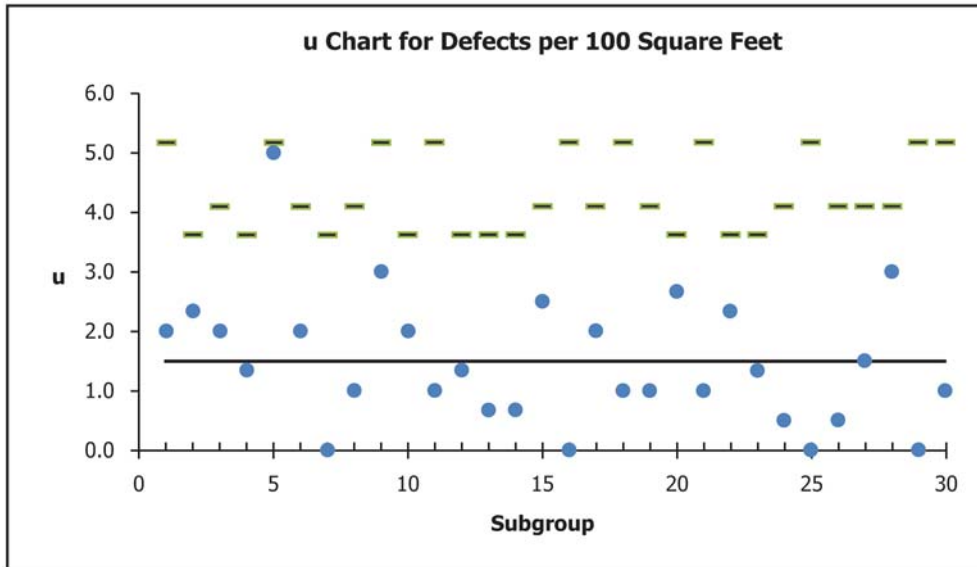
FIG. 12 *c* Chart for Monthly Minor Injuries

FIG. 13 *u* Chart for Defects per 100 Square Feet

determined empirically (**9, 14**) in two ways: (*1*) minimization of the forecasting errors ($Y_i − Z_{i-1}$) in the data set, or (*2*) choosing a value that maximizes the chance of detecting certain sizes of shifts in the average.

11.2 *Control Chart Setup and Calculations:*

11.2.1 The control limits for the EWMA chart depend on the stage of the SPC program (see 4.4). In Stage A there are little or no historical process estimates so the EWMA control limits must compensate for limited initial data, but in Stages B and C the EWMA can utilize historical estimates from previous stages and no such compensation is necessary.

11.2.1.1 The calculation of the EWMA for the first time point, Z_1, needs an initial EWMA value Z_0 to combine with the subgroup statistic Y_1. In Stage A, Z_0 can either be (*1*) set equal to Y_1, (*2*) to the average of the subgroup statistics in the initial set of data to be plotted, or (*3*) can be back-forecasted by conducting the EWMA in reverse and Z_0 will be the last value in the reversed EWMA series (**14**). In subsequent stages, Z_0 can be the last EWMA from the preceding stage. The effect of the starting value Z_0 on the EWMA dissipates rapidly with time.

The Z_0 value is plotted as the center line on the EWMA chart and is usually depicted as a solid line.

11.2.1.2 The successive EWMA values are calculated in time order using the current subgroup statistic and previous EWMA in (Eq 48). Plot the EWMA values as dots on the chart or as a connected line if the EWMA is plotted together with the companion statistic.

11.2.1.3 When historical information is available calculate the standard error of Z_t:

$$s_z = \hat{\sigma} \sqrt{\frac{\lambda}{(2 − \lambda)}} \qquad (49)$$

where $\hat{\sigma}$ depends on the subgroup statistic of the companion chart as follows:

(*1*) Averages from the *X*-bar, *R* chart, use equation (Eq 9) in 6.2.4,

(*2*) Averages from the *X*-bar, *s* chart, use equation (Eq 18) in 7.2.4,

(*3*) Observations from the *I, MR* chart, use equation (Eq 26) in 8.2.4,

17
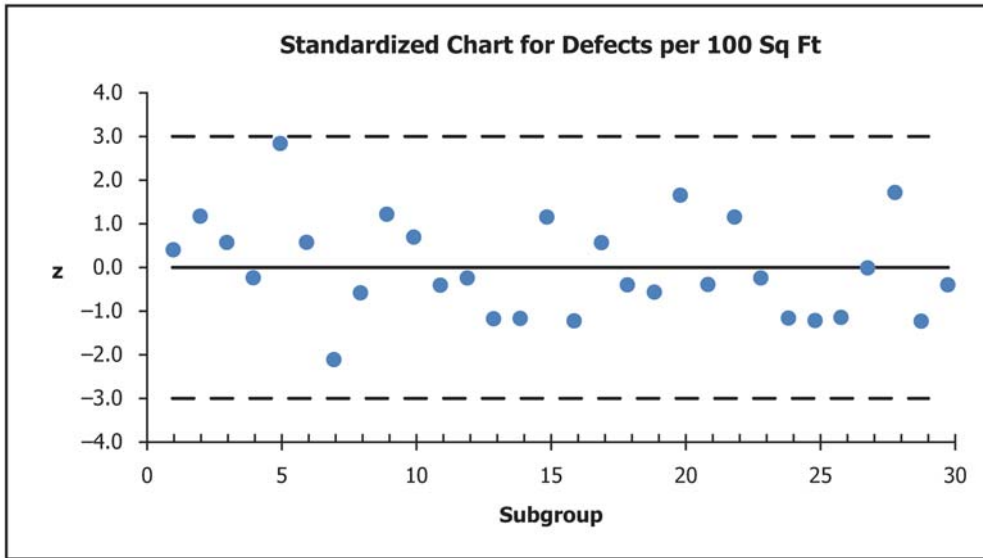
FIG. 14 Standardized Chart for Defects per 100 Square Feet
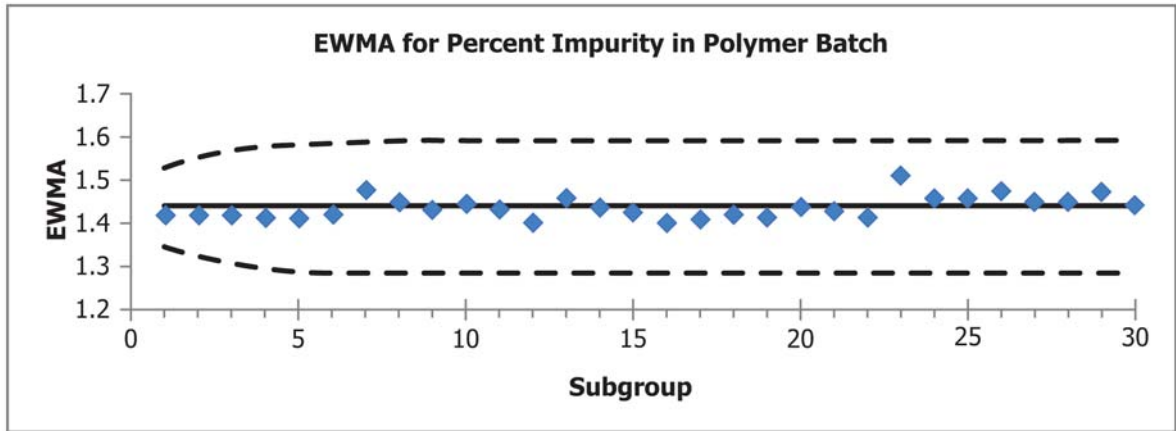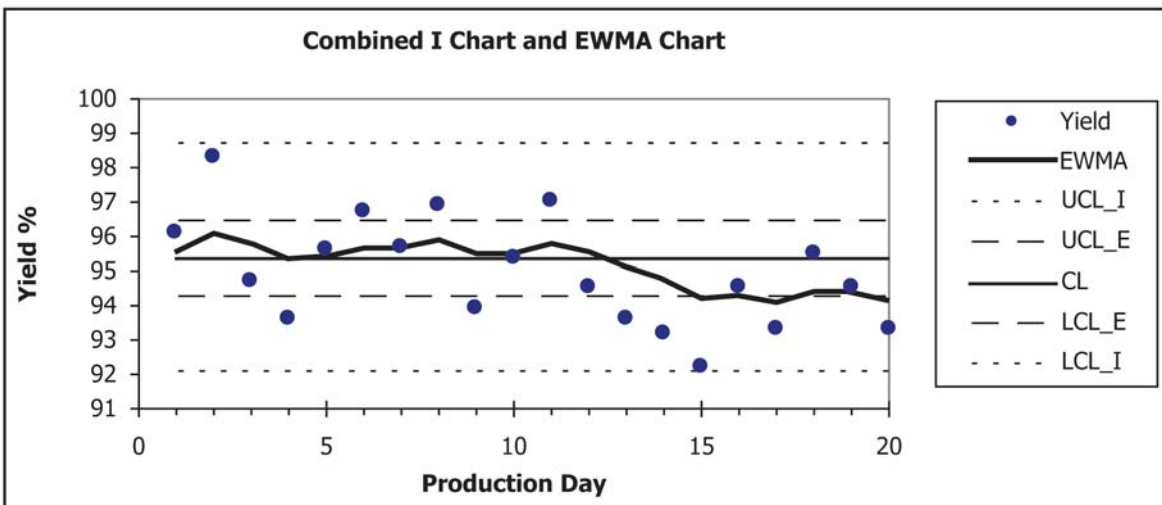

FIG. 15 EWMA Chart for Process Impurity

FIG. 16 Combined *I* Chart and EWMA Chart for Process Yield

(4) Fraction occurrences from the *p* chart, use equation (Eq 30) in 9.2.2.4,

(5) Numbers of occurrences from the *np* chart, use *n* times equation (Eq 30), and

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TABLE 8 Example of *u* Chart and Standardized Chart for Defects per 100 Square Feet of Fabric

Subgroup	Area	*n*	*c*	*u*	CL	UCL	*z*
1	100	1	2	2.0	1.5	5.2	0.4
2	300	3	7	2.3	1.5	3.6	1.2
3	200	2	4	2.0	1.5	4.1	0.6
4	300	3	4	1.3	1.5	3.6	−0.2
5	100	1	5	5.0	1.5	5.2	2.9
6	200	2	4	2.0	1.5	4.1	0.6
7	300	3	0	0.0	1.5	3.6	−2.1
8	200	2	2	1.0	1.5	4.1	−0.6
9	100	1	3	3.0	1.5	5.2	1.2
10	300	3	6	2.0	1.5	3.6	0.7
11	100	1	1	1.0	1.5	5.2	−0.4
12	300	3	4	1.3	1.5	3.6	−0.2
13	300	3	2	0.7	1.5	3.6	−1.2
14	300	3	2	0.7	1.5	3.6	−1.2
15	200	2	5	2.5	1.5	4.1	1.2
16	100	1	0	0.0	1.5	5.2	−1.2
17	200	2	4	2.0	1.5	4.1	0.6
18	100	1	1	1.0	1.5	5.2	−0.4
19	200	2	2	1.0	1.5	4.1	−0.6
20	300	3	8	2.7	1.5	3.6	1.6
21	100	1	1	1.0	1.5	5.2	−0.4
22	300	3	7	2.3	1.5	3.6	1.2
23	300	3	4	1.3	1.5	3.6	−0.2
24	200	2	1	0.5	1.5	4.1	−1.2
25	100	1	0	0.0	1.5	5.2	−1.2
26	200	2	1	0.5	1.5	4.1	−1.2
27	200	2	3	1.5	1.5	4.1	0.0
28	200	2	6	3.0	1.5	4.1	1.7
29	100	1	0	0.0	1.5	5.2	−1.2
30	100	1	1	1.0	1.5	5.2	−0.4
Total	6000	60	90				

(6) Counts of occurrences from the *c* chart, use equation (Eq 40) in 10.2.1.2.

11.2.1.4 When no historical information is available, the initial EWMA values are based on very little data and the exact equation for the standard error of the EWMA must be used, which is:

$$s_{z_i} = \hat{\sigma} \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \qquad (50)$$

for the *i*th subgroup. The term inside the brackets diminishes rapidly over time and the equation eventually converges to (Eq 49).

11.2.2 Calculate the lower control limit (LCL) and the upper control limit (UCL).

11.2.2.1 For the EWMA chart:

$$LCL = Z_0 - 3s_z \qquad (51)$$

$$UCL = Z_0 + 3s_z \qquad (52)$$

11.2.2.2 Control limits are depicted as dashed lines on the control charts.

11.2.3 EWMA statistics falling outside the control limits on the control chart may indicate the presence of a special cause requiring further investigation.

11.3 *Example*—The EWMA chart is used as a supplementary chart to the *I* chart (see 8.3) used for initial evaluation of the impurity level of a batch polymer process. The *I* chart in Fig. 6 indicated a single out-of-control value at Subgroup (Batch) 23. Assuming that the data listed in Table 4 was the initial data set in Stage A of this SPC project, there was no historical data available. Hence the starting EWMA value Z_0

used the average of the 30 batches which was equal to 1.437 %, and the EWMA values are calculated from (Eq 48) with λ = 0.2 and listed in Table 9.

11.3.1 The control limits were calculated by equation (Eq 50) and also appear in Table 9. The control limits appear closer to the center line for the initial subgroups, and converge to the values that would be calculated by equation (Eq 49). The EWMA chart shown in Fig. 15 gave no indication of an out-of-control situation at Subgroup 23 since the out-of-control signal was a transitory shift in level (a spike). The EWMA does not react as quickly as the *I* chart to this type of special cause variation.

11.4 *Example of Combined EWMA and I Charts*—Process yields are tracked daily on a five day per week operation. A successful SPC program had been conducted on this process, bringing the average yield up to 95 %, and the program was now in Stage C, or monitoring mode, using a combined *I* chart and a EWMA chart with λ = 0.2. Table 10 lists the yields (Y_i) for the current month of production, the calculated EWMA values (Z_i) and the moving ranges (MR_i). For this chart the historical average yield of 95.4 % was used for the starting EWMA (Z_0) and the center line of the control chart. The historical moving range of 1.24 % was used to estimate $\hat{\sigma}$ for calculating the EWMA control limits.

11.4.1 The control limits are calculated as follows:

11.4.1.1 *I Chart* (see 8.3.1.1):

$$LCL = 95.4 - (2.66)(1.24) = 92.1$$

$$UCL = 95.4 + (2.66)(1.24) = 98.7$$

11.4.1.2 *MR Chart (Not Plotted):*

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TABLE 9 Example of EWMA Chart for Polymer Impurities

Subgroup	Impurity	MR	EWMA	LCL	CL	UCL
1	1.39		1.428	1.345	1.437	1.530
2	1.42	0.03	1.426	1.319	1.437	1.556
3	1.42	0.00	1.425	1.305	1.437	1.570
4	1.39	0.03	1.418	1.297	1.437	0.578
5	1.40	0.01	1.414	1.292	1.437	1.583
6	1.46	0.06	1.424	1.288	1.437	1.586
7	1.70	0.24	1.479	1.286	1.437	1.588
8	1.33	0.37	1.449	1.285	1.437	1.589
9	1.36	0.03	1.431	1.284	1.437	1.590
10	1.50	0.14	1.445	1.284	1.437	1.591
11	1.38	0.12	1.432	1.284	1.437	1.591
12	1.28	0.10	1.402	1.283	1.437	1.591
13	1.68	0.40	1.457	1.283	1.437	1.591
14	1.35	0.33	1.436	1.283	1.437	1.592
15	1.38	0.03	1.425	1.283	1.437	1.592
16	1.30	0.08	1.400	1.283	1.437	1.592
17	1.44	0.14	1.408	1.283	1.437	1.592
18	1.47	0.03	1.402	1.283	1.437	1.592
19	1.38	0.09	1.412	1.283	1.437	1.592
20	1.54	0.16	1.438	1.283	1.437	1.592
21	1.38	0.16	1.426	1.283	1.437	1.592
22	1.34	0.04	1.409	1.283	1.437	1.592
23	1.91	0.57	1.509	1.283	1.437	1.592
24	1.24	0.67	1.455	1.283	1.437	1.592
25	1.45	0.21	1.454	1.283	1.437	1.592
26	1.55	0.10	1.473	1.283	1.437	1.592
27	1.35	0.20	1.449	1.283	1.437	1.592
28	1.45	0.10	1.449	1.283	1.437	1.592
29	1.56	0.11	1.471	1.283	1.437	1.592
30	1.32	0.24	1.441	1.283	1.437	1.592
Average	1.437	0.165				
Standard deviation		0.146				

TABLE 10 Combination EWMA and *I* Chart Example for Process Yield

Day	Yield	EWMA	MR		
1	96.1	95.5	...		
2	98.3	96.1	2.2	*Historical Data*	
3	94.7	95.8	3.6	Average	95.4
4	93.6	95.4	1.1	MRBAR	1.24
5	95.6	95.4	2.0		
6	96.7	95.7	1.1	*I Chart Limits*	
7	95.7	95.7	1.0	UCL	98.7
8	96.9	95.9	1.2	LCL	92.1
9	93.9	95.5	3.0		
10	95.4	95.5	1.5	*EWMA Chart Limits*	
11	97.0	95.8	1.6	UCL	96.5
12	94.5	95.5	2.5	LCL	92.1
13	93.6	95.1	0.9		
14	93.2	94.8	0.4	*Center Line*	
15	92.2	94.2	1.0	CL	95.4
16	94.5	94.3	2.3		
17	93.3	94.1	1.2	*MR Chart Limits*	
18	95.5	94.4	2.2	UCL	4.1
19	94.5	94.4	1.0	LCL	0.0
20	93.3	94.2	1.2		

$$UCL = (3.27)(1.24) = 4.1$$

11.4.1.3 *EWMA Chart:*

$$\hat{\sigma} = 1.24/1.128 + 1.099$$

$$s_z = 1.099\sqrt{0.2/(2 - 0.2)} + (1.099)\sqrt{1/3} = 0.366$$

$$LCL = 95.4 - (3)(0.366) = 94.3$$

$$UCL = 95.4 + (3)(0.366) = 96.5$$

It is noteworthy that EWMA control limits are equivalent to the one-sigma limits for the *I* chart when $\lambda = 0.2$.

11.4.2 The *I* chart and EWMA control charts are plotted together (Fig. 16) with the *I* chart control limits depicted as dotted lines and the EWMA chart control limits depicted as dashed lines.

11.4.2.1 The *I* chart did not indicate a shift, even though there appeared to be a decrease in yield starting around Day 12 or 13. If the Western Electric rules were applied for detections of a shift (5.2.2.1), a shift would also not have been detected:

(1) The lower warning limit, midway between the *I* chart LCL and EWMA chart LCL, is 93.2, and no two consecutive points fall below this limit,

(2) There are no cases where four out of five consecutive points fall below the lower one-sigma limit of the *I* chart (equal to the EWMA lower control limit), and

(3) There are no cases of eight consecutive points below the center line. There were also no moving ranges above the UCL for the *MR* chart (not plotted).

11.4.2.2 The EWMA chart indicated out-of-control points at Days 15, 17, and 20. The downward shift appeared to be on the order of 1 % yield or about one standard deviation. An investigation was conducted to find the special cause of the lower yields.

12. Control Chart Using Cumulative Sum (CUSUM) Chart

12.1 *Control Chart Usage*—The CUSUM chart uses the cumulative sum of the deviations of the sample values from process target value. Cumulative sum is a running sum of deviations from a preselected target value. The CUSUM chart is very sensitive to slow drifts in process mean because steady minor deviation from the process mean will result in a steady increase or decrease of cumulative deviations.

12.1.1 Denote by Y_j the subgroup statistic for the *j*th subgroup. This statistic may be individual observations, subgroup averages, percentages, or counts. Let *T* denote the process target value. The CUSUM statistic at time, I, is denoted as C_i and is calculated as:

$$C_i = \sum_{j=1}^{i}(Y_j - T) \tag{53}$$

12.1.1.1 Because the cumulative sum, C_i, combines information from previously collected subgroups, a CUSUM chart is more effective than an I-chart or an *X*-bar chart to detect small shifts in process. CUSUM charts are especially effective for subgroups of Size 1.

12.1.1.2 *Selecting Target Value*—The process target value is chosen depending on the process purpose. It can be a historical mean value for control material or an accepted reference value (ARV) for tested material. In some cases, a target value may vary over time as, for example, when differing lots of a material are being monitored and each has a different mean for the characteristic being monitored.

12.1.2 A common method for presenting the CUSUM control chart is referred to as the tabular method. This separately monitors deviations above and below the process target. A less popular CUSUM charting method, the V-mask method, is not covered in this practice.

12.1.3 In the tabular CUSUM control chart, the cumulative sum of deviations above the process target mean and exceeding

the allowance value is presented as the upper CUSUM. The cumulative sum of deviations below the target and exceeding the allowance value is presented as the lower CUSUM. Both are plotted on the chart. For an in-control process, points fluctuate above and below the center line. A downward trend of points in a lower CUSUM or an upward trend of points in an upper CUSUM are indications that the process mean is shifting. The upper CUSUM detects upward trends in process mean, while the lower CUSUM detects downward trends in process mean.

12.1.4 The upper CUSUM is denoted as C^+ and the lower CUSUM as C^-. They are calculated as:

$$C_i^+ = \max[0\ ,\ C_{i+1}^+ + Y_i\ -\ (T + K)] \qquad (54)$$

$$C_i^- = \min[0\ ,\ C_{i-1}^- + Y_i\ -\ (T - K)] \qquad (55)$$

where:
Y_i = subgroup statistic at time, i, and
K = allowance value.

12.1.5 The CUSUM design is specified by the parameters, H and K. The decision interval, H, is the distance between the center line and control limits and is defined by a multiplier (h) of standard deviation:

$$H = h \cdot \hat{\sigma} \qquad (56)$$

12.1.5.1 The allowance value, K, is the amount of process shift to be detected and is defined as:

$$K = k \cdot \hat{\sigma} \qquad (57)$$

$$k = \frac{1}{2} \cdot \delta \qquad (58)$$

where:
$\hat{\sigma}$ = estimated short-term standard deviation, and
δ = relative size of the process shift to detect in standard deviation units.

12.1.5.2 By choosing values, k and h, we tune the chart's sensitivity to detect small shifts in terms of average run length (ARL) performance. The most common CUSUM chart designs have parameter, k, equal to 0.5 and parameter, h, equal to 4 or 5. These two designs are very efficient in quickly detecting a 1σ process shift. More discussion on selecting CUSUM design and on CUSUM chart ARL performance is found in Montgomery (**4**).

12.2 *Tabular CUSUM Control Chart Setup and Calculation:*

12.2.1 Define target value for tested control material and control chart design parameters, H and K.

12.2.1.1 Calculate short-term standard deviation $\hat{\sigma}$ from an I-chart or an X-bar chart as one-third of the difference between the upper control limit and the center line value.

12.2.2 Starting values for tabular CUSUM control chart are:

$$C_0^- = C_0^+ = 0 \qquad (59)$$

12.2.2.1 Successive CUSUM C^+ and C^- values are calculated in time order for each subgroup using the current subgroup statistics, the previous C^+ and C^- values, the chosen process target, T, and the allowance value, K, as described in Eq 54 and Eq 55.

12.2.2.2 Plot calculated CUSUM C^+ and C^- values as dots connected with a line on control chart.

12.2.3 Place the center line for the CUSUM chart at 0.

12.2.4 Control limits are defined with decision interval H and subgroup size n:

$$LCL = -h \frac{\hat{\sigma}}{\sqrt{n}} \qquad (60)$$

$$UCL = h \frac{\hat{\sigma}}{\sqrt{n}} \qquad (61)$$

12.2.4.1 Plot control limits as dashed lines on the CUSUM chart.

12.2.5 CUSUM statistics C_i^+ or C_i^- falling outside the control limits on the control chart indicate the presence of assignable cause that needs to be investigated and addressed.

12.2.5.1 The Western Electric run rules do not apply to CUSUM charts because the C_i^+ and C_i^- statistics are not independent over time.

12.2.5.2 The CUSUM chart helps to identify the initial appearance of the assignable cause that generated the out-of-control signal. It is identified by looking backwards from the first out-of-control signal to the subgroup time when the CUSUM (upper or lower, depending on the direction of the out of control signal) first departed from the center line.

12.3 *Example—Petroleum Distillate Temperature*—Boiling range characteristics of refinery products are determined by distillation of petroleum at atmospheric pressure. Table 11 presents Stage C (process-monitoring) individual data for testing quality control sample ($n = 1$) of a refinery product. The target boiling temperature value for this product is 493°F. Fig. 17 presents an I-chart for Stage C with mean temperature 494.3°F and short-term standard deviation 1.01°F. Points 11 to 20 on the I-chart are all below the center line, which indicates that the process is below the mean value of 494.3°F. The CUSUM chart with design parameters, $h = 4$ and $k = 0.5$, is used to evaluate whether the process is at the target level of 493°F. The CUSUM chart is presented in Fig. 18.

12.3.1 The control limits for the CUSUM chart were calculated as follows:

$$LCL = -4 \cdot 1.01 = -4.04 \qquad (62)$$

$$UCL = 4 \cdot 1.01 = 4.04 \qquad (63)$$

12.3.2 Table 11 presents control chart parameters and limits and the CUSUM C^+ and C^- values. Points 1 to 23 on the CUSUM chart are within control limits. This indicates that the process mean was at the target of 493°F. Points 21 to 26 are steadily increasing with Points 24 to 26 rising above the upper control limit. This indicates an upward shift in the process mean. At Point 24, the process is considered to be out of control. Following the points backwards from Point 24, Point 21 is the beginning of the increasing pattern. This identifies a likely candidate time for the process shift above the target level.

13. Control Chart Using the Exponentially-Weighted Moving Variance (EWMV) Chart

13.1 *Control Chart Usage*—The EWMV control chart uses the exponentially-weighted moving variance (EWMV)

TABLE 11 Example of I-Chart and CUSUM Chart for Petroleum Distillate Temperature

		I-Chart Limits			CUSUM Chart				
Sample	Result	LCL	UCL	LC	C^+	C^-	LCL	UCL	CL
1	493.5	491.3	497.3	494.3	0.00	0.00	−4.04	4.04	0
2	493.5	491.3	497.3	494.3	0.00	0.00	−4.04	4.04	0
3	495.1	491.3	497.3	494.3	1.60	0.00	−4.04	4.04	0
4	492.0	491.3	497.3	494.3	0.09	−0.50	−4.04	4.04	0
5	494.6	491.3	497.3	494.3	1.19	0.00	−4.04	4.04	0
6	493.6	491.3	497.3	494.3	1.28	0.00	−4.04	4.04	0
7	494.2	491.3	497.3	494.3	1.98	0.00	−4.04	4.04	0
8	492.1	491.3	497.3	494.3	0.57	−0.39	−4.04	4.04	0
9	493.5	491.3	497.3	494.3	0.57	0.00	−4.04	4.04	0
10	495.4	491.3	497.3	494.3	2.46	0.00	−4.04	4.04	0
11	492.9	491.3	497.3	494.3	1.86	0.00	−4.04	4.04	0
12	492.9	491.3	497.3	494.3	1.25	0.00	−4.04	4.04	0
13	493.4	491.3	497.3	494.3	1.15	0.00	−4.04	4.04	0
14	493.8	491.3	497.3	494.3	1.44	0.00	−4.04	4.04	0
15	493.4	491.3	497.3	494.3	1.34	0.00	−4.04	4.04	0
16	492.2	491.3	497.3	494.3	0.03	−0.30	−4.04	4.04	0
17	493.7	491.3	497.3	494.3	0.23	0.00	−4.04	4.04	0
18	493.4	491.3	497.3	494.3	0.12	0.00	−4.04	4.04	0
19	493.7	491.3	497.3	494.3	0.31	0.00	−4.04	4.04	0
20	493.3	491.3	497.3	494.3	0.11	0.00	−4.04	4.04	0
21	494.3	491.3	497.3	494.3	0.91	0.00	−4.04	4.04	0
22	495.7	491.3	497.3	494.3	3.10	0.00	−4.04	4.04	0
23	493.8	491.3	497.3	494.3	3.40	0.00	−4.04	4.04	0
24	495.9	491.3	497.3	494.3	5.79	0.00	−4.04	4.04	0
25	495.9	491.3	497.3	494.3	8.19	0.00	−4.04	4.04	0
26	494.9	491.3	497.3	494.3	9.58	0.00	−4.04	4.04	0
Process mean		494.3							
Short-term standard deviation		1.01							
Process target		493.0							
k		0.5							
h		4							
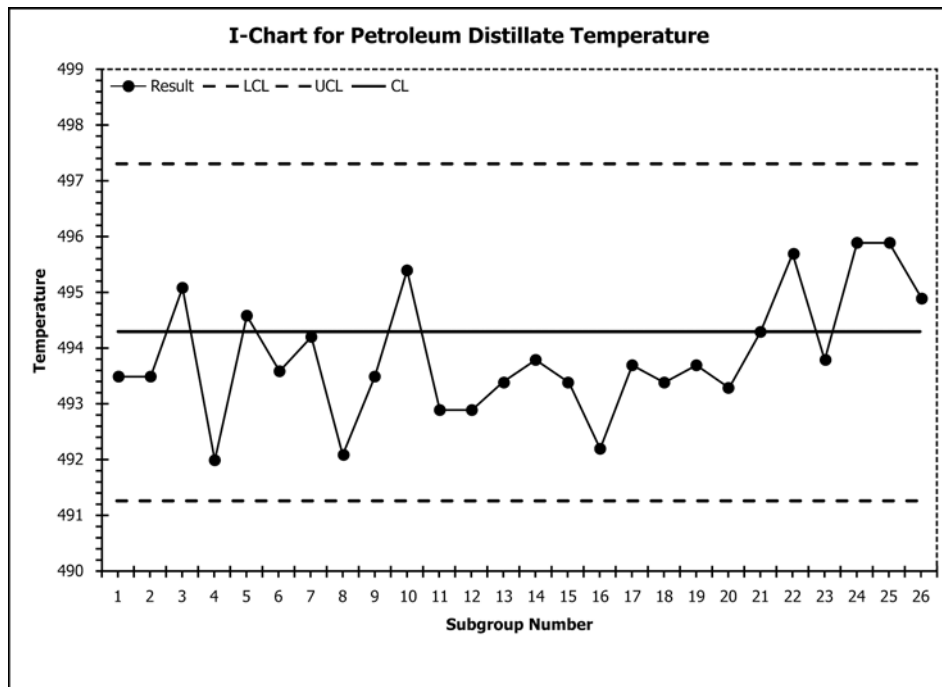
FIG. 17 I-Chart for Petroleum Distillate Temperature

statistic, a type of weighted average of the current and past squared deviations of observations minus their estimated process averages. The squared deviation at time i is:

$$D_i^2 = (Y_i - Z_{i-1})^2 \qquad (64)$$

where Y_i is the single subgroup observation at time i, and Z_{i-1} is the forecasted value of the process average estimated from the EWMA statistic at time $i–1$ from a companion
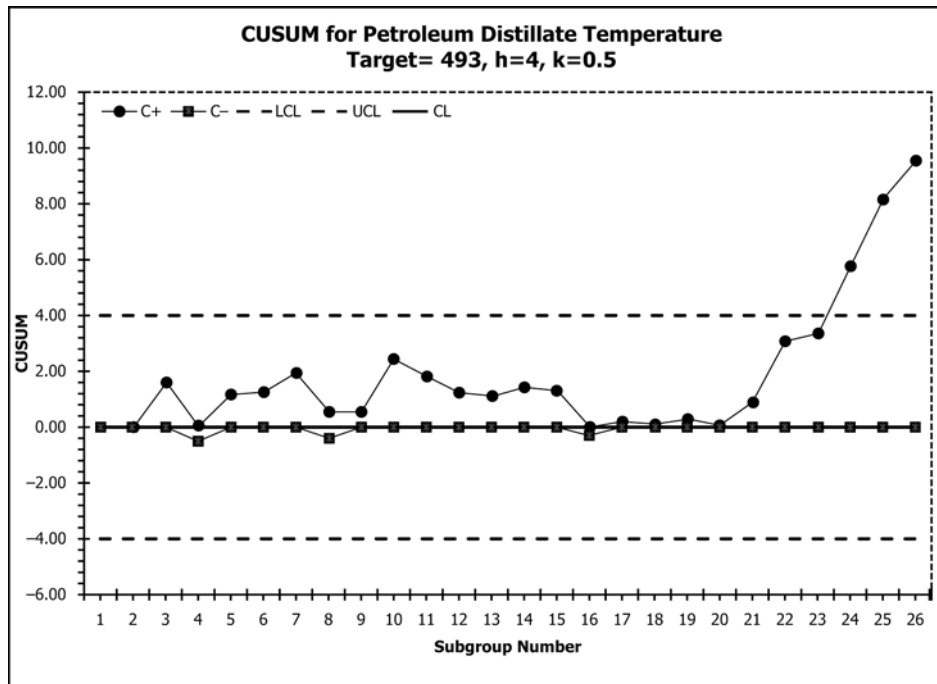
FIG. 18 CUSUM Chart for Petroleum Distillation Temperature

EWMA chart (see 11.1.1). The EWMV statistic at time i is denoted as V_i, and is calculated recursively as:

$$V_i = \omega D_i^2 + (1 - \omega)V_{i-1} \tag{65}$$

where V_{i-1} is the EWMV at the preceding time point $i–1$ and ω (omega) is a weighting factor ($0 < \omega < 1$). The weighting factor ω is used to control the rate of decrease of the weights with time, similar to the weighting factor λ used for the EWMA chart.

13.1.1 Fluctuations in the process mean will also affect the variance estimate. Use of the companion EWMA chart to estimate the current process mean is a convenient way to reduce this effect.

NOTE 10—Another form of a variance chart uses a constant value for the process average based on previous in-control information. That is, a known standard μ_0 is substituted for Z_{i-1} in the equation (Eq 65) above. This chart has been discussed by MacGregor and Harris (**17**) and was termed by them as an Exponentially-Weighted Mean Square Chart (EWMS Chart). The EWMS chart will react to changes in the process mean as well as the process variance and is therefore not specific for monitoring variances.

13.1.2 The EWMV statistic at time i is a weighted sum of the current and past squared deviations:

$$V_i = \omega D_i^2 + \omega(1 - \omega)D_{i-1}^2 + \omega(1 - \omega)^2 D_{i-2}^2 + ... \tag{66}$$

thus for independent observations Y_i with variance σ, it can be shown (**17**) that V_i / σ^2 is approximately distributed as $\chi^2(\nu)/\nu$, with effective degrees of freedom:

$$\nu = (2 - \omega)/\omega \tag{67}$$

13.1.3 The choice for the value of the weighting factor ω is at the discretion of the user, and the effective degrees of freedom for the EWMV statistic from Eq 66 corresponding to a given value of the weighting factor is tabulated below for selected values of ω.

EMWV Factor, ω	Degrees of Freedom, ν
0.01	199
0.02	99
0.05	39
0.10	19
0.20	9
0.33	5
0.40	4
0.50	3

Smaller values of ω are associated with higher effective degrees of freedom of the EWMV and allow the estimate of the variance to reach back in time with a sample size roughly equivalent to the effective degrees of freedom. For processes with a high frequency of data collection, EMWV factors in the range of 0.01–0.05 give long term estimates of the process variance. For processes having less frequent data collection, EWMV factors in the range 0.05–0.2 may be used for intermediate term variance estimates. The factor range of 0.33–0.5 provides short-term variance estimates akin to those given by a Range Chart with subgroup sizes of $n \leq 6$. The use of a value of ω greater than 0.5 is uncommon, because the EWMV would then be based on very small degrees of freedom with little weight given to the past data.

13.2 *Control Chart Setup and Calculations:*

13.2.1 Initial estimates of the process average (Z_0) and process variance (V_0) are necessary to start the calculation of the EWMV. For general guidance on obtaining estimates of Z_0, see 11.2.1. An estimate of V_0 is provided by data from an in-control period of the process. The effect of these starting values dissipates with time. Calculate the successive V_i values (Eq 65) for each subgroup (time) and plot these as dots versus the subgroup number.

13.2.2 The EWMA and EWMV charts are said to be coupled, as the EWMA chart provides continuing information on the estimated process mean for use in the EWMV chart. The

EWMV control chart limits are provided by the methodology in Sweet (**18**), which are Shewhart (3σ) limits, assuming an independent normal distribution for the variation in the monitored process variable. These limits are given in Table 12 and are dependent on the values of the chosen values of the EWMA weighting factor λ and the EWMV weighting factor ω. Table 12 also lists the effective degrees of freedom for the EWMV statistic as discussed in 13.1.3.

13.2.3 Use the V_0 value and the EWMA weighting factor λ to calculate the center line, which is usually depicted as a solid line on the chart.

$$CL = [2 / (2 - \lambda)]V_0 \qquad (68)$$

13.2.4 From Table 12, find the control chart factors B_5^* and B_6^* for chosen values of the EWMA weighting factor λ and the EWMV weighting factor ω. Calculate the lower and upper control limits for the EWMV chart as follows:

$$LCL = B_5^* V_0 \qquad (69)$$

$$UCL = B_6^* V_0 \qquad (70)$$

The control chart limits are usually depicted as dashed lines on the chart. A value of the EWMV outside these control limits signals a shift in the variance.

13.3 *Example*—Continuing with the polymer impurity data example from 8.2.3 and 11.3, an initial value of the process variance was calculated from the in-control data of subgroups

(batches) 1–22, giving the estimate $V_0 = 0.0113$. The EWMA chart had a weighting factor $\lambda = 0.2$, and a weighting factor $\omega = 0.05$ was chosen for the EWMV chart to monitor the long term variance. The EWMV control chart factors corresponding to these weighting factor values are given from Table 12 as $B_5^* = 0.323$ and $B_6^* = 1.884$. The control chart center line and control limits were calculated as:

$$CL = [2 / (2 - 0.2)](0.01127) = (1.11111)(0.01127) = 0.01252$$
$$LCL = (0.323)(0.01127) = 0.00364$$
$$UCL = (1.884)(0.01127) = 0.02123$$

The impurity data Y_i, the forecasted average Z_{I-1}, the Squared Deviation D_i^2, and the EWMV V_i are listed for each subgroup in Table 13. The EWMV chart is shown in Fig. 19, and indicated an upward shift in the variance at subgroup 23.

13.3.1 Table 12 also gives the effective degrees of freedom for the EWMA statistic with weighting factor ω using the calculation in Eq 67. For $\omega = 0.05$ in this example, $\nu = 39$; therefore, this EWMV chart is monitoring the long term variability of the process.

14. Keywords

14.1 common cause; control chart; control limits; rational subgroup; special cause; state of statistical control; statistical process control

TABLE 12 Control Chart Factors for EWMV Chart Given the EWMV and EMWA Factors

EWMA Chart Wt. Factor, λ	Control Chart Factors	EWMA Chart Weighting Factor, ω							
		0.01	0.02	0.05	0.10	0.20	0.33	0.40	0.50
0.05	B_5^*	1.336	1.465	1.725	2.027	2.479	2.974	3.203	3.539
	B_6^*	0.715	0.586	0.326	0.024	0.000	0.000	0.000	0.000
0.10	B_5^*	1.373	1.507	1.775	2.086	2.550	3.057	3.292	3.636
	B_6^*	0.732	0.598	0.330	0.020	0.000	0.000	0.000	0.000
0.15	B_5^*	1.413	1.551	1.828	2.148	2.626	3.135	3.388	3.741
	B_6^*	0.750	0.611	0.334	0.014	0.000	0.000	0.000	0.000
0.20	B_5^*	1.454	1.597	1.884	2.215	2.708	3.244	0.000	0.000
	B_6^*	0.768	0.625	0.323	0.007	0.000	0.000	0.000	0.000
0.25	B_5^*	1.498	1.647	1.944	2.286	2.795	3.347	3.601	3.973
	B_6^*	0.787	0.639	0.267	0.000	0.000	0.000	0.000	0.000
0.30	B_5^*	1.545	1.699	2.007	2.362	2.889	3.458	3.719	4.101
	B_6^*	0.808	0.654	0.346	0.000	0.000	0.000	0.000	0.000
0.35	B_5^*	1.595	1.755	3.075	2.443	2.989	3.576	3.846	4.238
	B_6^*	0.829	0.669	0.350	0.000	0.000	0.000	0.000	0.000
0.40	B_5^*	1.648	1.814	2.147	2.530	3.096	3.704	3.982	4.386
	B_6^*	0.852	0.686	0.338	0.000	0.000	0.000	0.000	0.000
0.50	B_5^*	1.766	1.946	2.307	2.722	3.333	3.987	4.284	4.714
	B_6^*	0.901	0.720	0.360	0.000	0.000	0.000	0.000	0.000
Effective EWMV df		199	99	39	19	9	5	4	3

TABLE 13 Example of EWMV Chart for Polymer Impurity

Time	Y_i	Z_{i-1}	D_i^2	V_i
1	1.39	1.428	0.0010	0.0108
2	1.42	1.426	0.0001	0.0102
3	1.42	1.425	0.0000	0.0097
4	1.39	1.418	0.0012	0.0093
5	1.40	1.414	0.0003	0.0088
6	1.46	1.424	0.0021	0.0085
7	1.70	1.479	0.0764	0.0119
8	1.33	1.449	0.0222	0.0124
9	1.36	1.431	0.0079	0.0122
10	1.50	1.445	0.0047	0.0118
11	1.38	1.432	0.0042	0.0114
12	1.28	1.402	0.0231	0.0120
13	1.68	1.457	0.0775	0.0153
14	1.35	1.436	0.0115	0.0151
15	1.38	1.425	0.0031	0.0145
16	1.30	1.400	0.0155	0.0146
17	1.44	1.408	0.0016	0.0139
18	1.47	1.420	0.0039	0.0134
19	1.38	1.412	0.0016	0.0128
20	1.54	1.438	0.0163	0.0130
21	1.38	1.426	0.0033	0.0125
22	1.34	1.409	0.0074	0.0124
23	1.91	1.509	0.2510	0.0242
24	1.24	1.455	0.0724	0.0266
25	1.45	1.454	0.0000	0.0253
26	1.55	1.473	0.0092	0.0245
27	1.35	1.449	0.0152	0.0240
28	1.45	1.449	0.0000	0.0228
29	1.56	1.471	0.0123	0.0223
30	1.32	1.441	0.0229	0.0223
LCL		0.0038		
CL		0.0125		
UCL		0.0212		
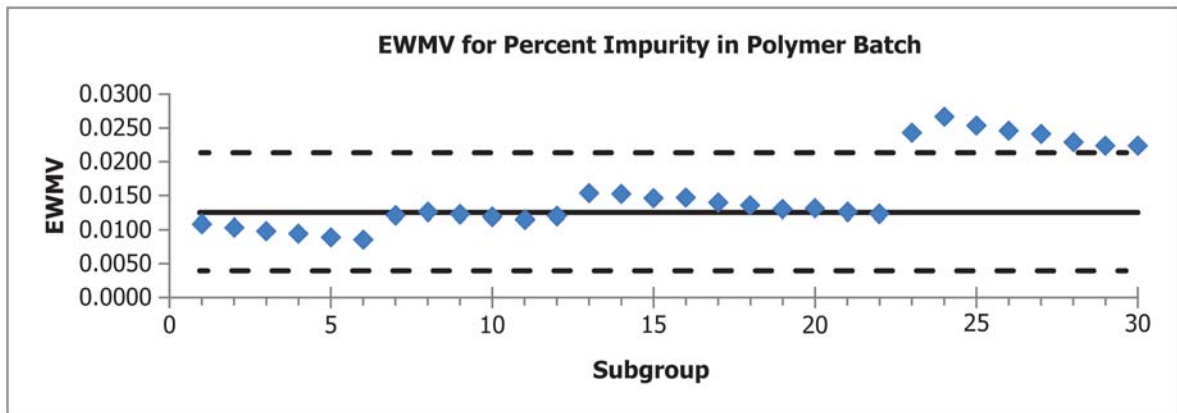
FIG. 19 EWMV Chart for Process Impurity

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APPENDIX

(Nonmandatory Information)

X1. EXAMPLES OF EWMA CHARTS AS SUPPLEMENTARY CHARTS TO SHEWHART CHARTS

X1.1 Companion Chart to *X*-Bar, R-Chart (See 6.3 Example—Liquid Product Filling into Bottles)

X1.1.1 At a frequency of 30 min, four consecutive bottles were pulled from the filling line and weighed. The observations, subgroup averages, and subgroup ranges are listed in Table 2, and the grand average and average range are calculated at the bottom of the table. The average weight, Y_i, the EWMA, Z_i, and the EWMA control limits are shown in Table X1.1.

X1.1.2 The control limits are calculated as follows:

X1.1.2.1 EWMA chart using $Z_0 = 246.44$ $s_z = 1.355$ and $\lambda = 0.2$:

TABLE X1.1 Example of Bottle-Filling Operation with EWMA (λ=0.2)

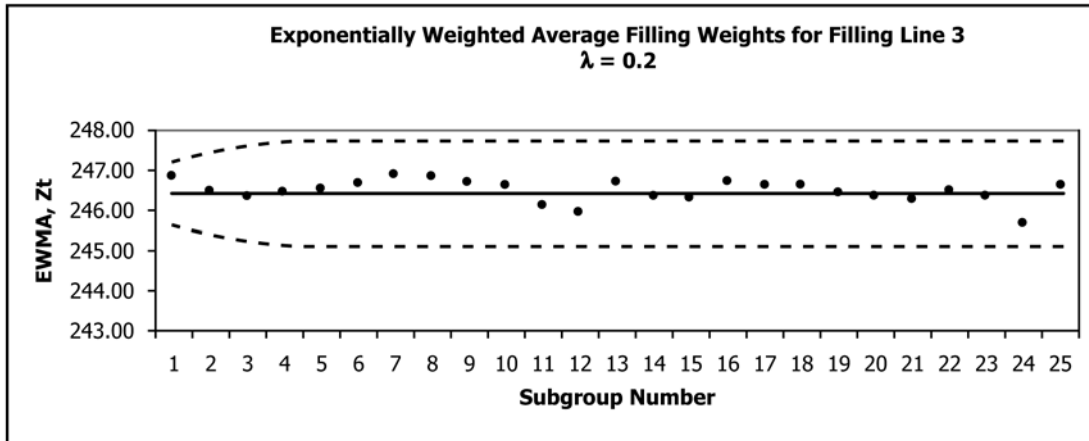| Subgroup	Average	EWMA	LCL	CL	UCL
1	248.38	246.83	245.63	246.44	247.25
2	244.98	246.46	245.40	246.44	247.48
3	245.85	246.32	245.28	246.44	247.60
4	246.88	246.43	245.20	246.44	247.68
5	246.70	246.49	245.16	246.44	247.72
6	247.35	246.66	245.13	246.44	247.75
7	248.58	246.87	245.11	246.44	247.77
8	246.58	246.81	245.10	246.44	247.78
9	247.58	246.69	245.10	246.44	247.78
10	246.58	246.60	245.09	246.44	247.79
11	244.73	246.10	245.09	246.44	247.79
12	245.28	245.93	245.09	246.44	247.79
13	247.70	246.69	245.09	246.44	247.79
14	244.80	246.31	245.09	246.44	247.79
15	245.78	246.31	245.09	246.44	247.79
16	248.20	246.69	245.09	246.44	247.79
17	247.23	246.60	245.09	246.44	247.79
18	246.70	246.62	245.08	246.44	247.80
19	246.33	246.42	245.08	246.44	247.80
20	246.05	246.34	245.08	246.44	247.80
21	245.55	246.26	245.08	246.44	247.80
22	247.23	246.46	245.08	246.44	247.80
23	245.90	246.33	245.08	246.44	247.80
24	243.03	245.67	245.08	246.44	247.80
25	247.23	246.60	245.08	246.44	247.80

$$LCL = 246.44 - 3\,(1.355)\,\sqrt{\frac{0.2}{2-0.2}}\sqrt{1-(1-0.2)^{2t}}$$

$$UCL = 246.44 + 3\,(1.355)\,\sqrt{\frac{0.2}{2-0.2}}\sqrt{1-(1-0.2)^{2t}}$$

X1.1.3 The control chart is shown in Fig. X1.1. The EWMA chart indicates that the filling weights are in statistical control.

X1.2 Companion Chart to *p* Chart (See 9.3 Example— Nonconforming Cartons)

X1.2.1 Cartons are inspected each shift in samples of 200 for minor (cosmetic) defects (such as tearing, dents, or scoring). From Table 5, Table X1.2 lists the number of nonconforming cartons (*X*) and the fraction defective (*p*) for 30 inspections. The average fraction defective was 0.058 and was used as the EWMA starting value Z_0, and the standard deviation $\acute{\sigma}$ was 0.0165 by (Eq 30). The EWMA was calculated by equation (Eq 48) and the EWMA control limits were calculated by equation (Eq 50) as listed in Table X1.2.

X1.2.2 The EMWA chart is shown in Fig. X1.2 and gave evidence of a lack of statistical control at subgroups 23 and 24.

X1.3 Companion Chart to *c* Chart (See 10.3 Example— Minor injuries)

X1.3.1 The number of minor injuries per month are tracked for a manufacturing plant with a stable workforce over a two-year period in Table 7. The average count was 3.3 and was used as the EWMA starting value Z_0, and the standard deviation $\acute{\sigma}$ was 1.82 by (Eq 40). The EWMA was calculated by (Eq 48) and the control limits were calculated by equation (Eq 50) as listed in Table X1.3.

X1.3.2 The EWMA chart is shown in Fig. X1.3 and indicated no lack of statistical control as was also indicated by the *c* chart.

FIG. X1.1 EWMA Chart for Filling Line 3

**TABLE X1.2 Example—Nonconforming Cartons with EWMA
(λ=0.4)**

| Subgroup	X	p	EWMA	LCL	CL	UCL
1	12	0.060	0.059	0.038	0.058	0.078
2	15	0.075	0.065	0.035	0.058	0.081
3	8	0.040	0.055	0.034	0.058	0.082
4	10	0.050	0.053	0.033	0.058	0.083
5	4	0.020	0.040	0.033	0.058	0.083
6	7	0.035	0.038	0.033	0.058	0.083
7	16	0.080	0.055	0.033	0.058	0.083
8	9	0.045	0.051	0.033	0.058	0.083
9	14	0.070	0.058	0.033	0.058	0.083
10	10	0.050	0.055	0.033	0.058	0.083
11	5	0.025	0.043	0.033	0.058	0.083
12	6	0.030	0.038	0.033	0.058	0.083
13	17	0.085	0.057	0.033	0.058	0.083
14	12	0.060	0.058	0.033	0.058	0.083
15	22	0.110	0.079	0.033	0.058	0.083
16	8	0.040	0.063	0.033	0.058	0.083
17	10	0.050	0.058	0.033	0.058	0.083
18	5	0.025	0.045	0.033	0.058	0.083
19	13	0.065	0.053	0.033	0.058	0.083
20	11	0.055	0.054	0.033	0.058	0.083
21	20	0.100	0.072	0.033	0.058	0.083
22	18	0.090	0.079	0.033	0.058	0.083
23	24	0.120	0.096	0.033	0.058	0.083
24	15	0.075	0.087	0.033	0.058	0.083
25	9	0.045	0.070	0.033	0.058	0.083
26	12	0.060	0.066	0.033	0.058	0.083
27	7	0.035	0.054	0.033	0.058	0.083
28	13	0.065	0.058	0.033	0.058	0.083
29	9	0.045	0.053	0.033	0.058	0.083
30	6	0.030	0.044	0.033	0.058	0.083
Average	11.6	0.058	λ	0.4		
Standard deviation	3.31	0.0165				
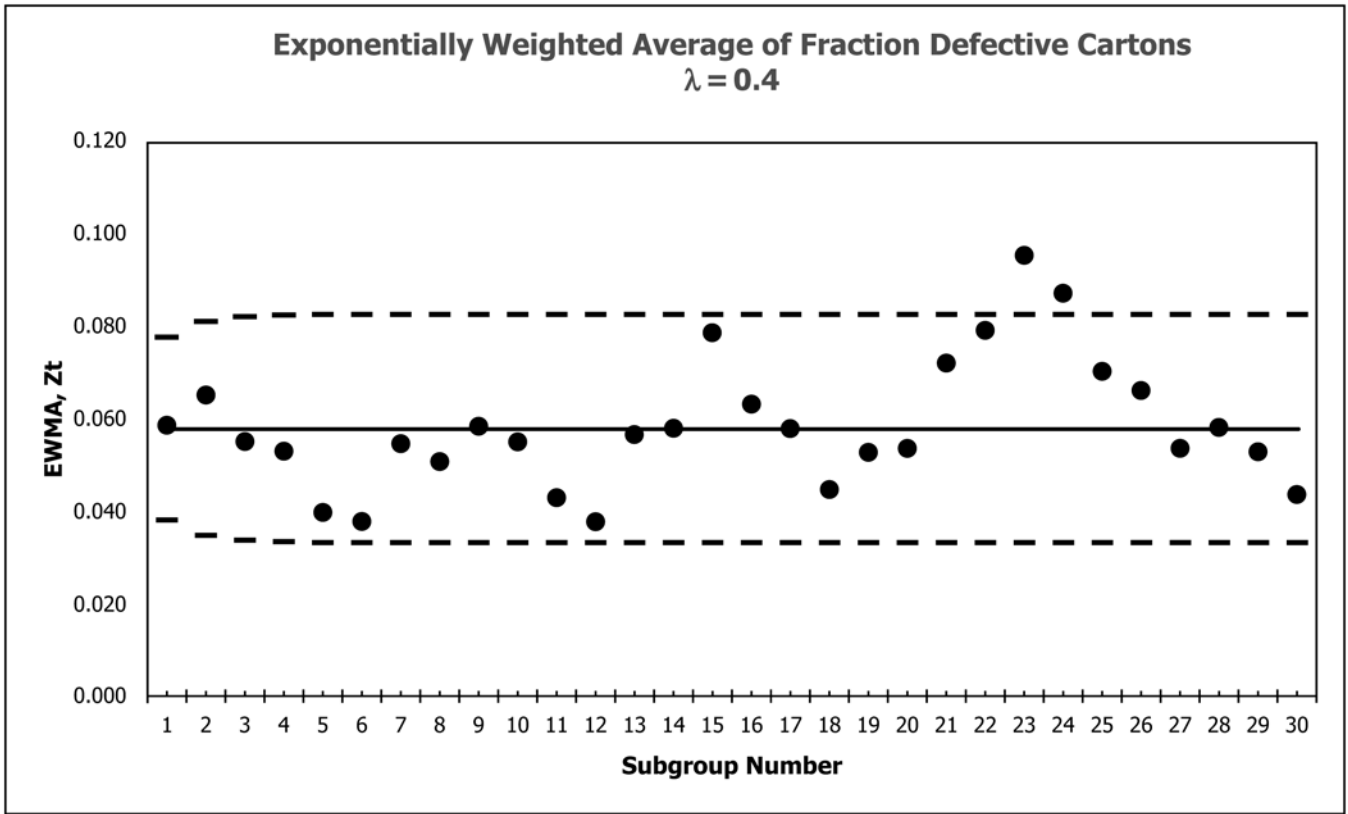
FIG. X1.2 EWMA Chart for Nonconforming Cartons

TABLE X1.3 EWMA Chart Calculations for Monthly Minor Injuries

| Month	c	EWMA	LCL	CL	UCL
1	5	4.0	1.1	3.3	5.5
2	3	3.6	0.8	3.3	5.8
3	2	3.0	0.6	3.3	6.0
4	4	3.4	0.6	3.3	6.0
5	0	2.0	0.6	3.3	6.0
6	3	2.4	0.6	3.3	6.0
7	4	3.0	0.6	3.3	6.0
8	1	2.2	0.6	3.3	6.0
9	4	2.9	0.6	3.3	6.0
10	8	5.0	0.6	3.3	6.0
11	3	4.2	0.6	3.3	6.0
12	1	2.9	0.6	3.3	6.0
13	0	1.7	0.6	3.3	6.0
14	4	2.6	0.6	3.3	6.0
15	6	4.0	0.6	3.3	6.0
16	2	3.2	0.6	3.3	6.0
17	4	3.5	0.6	3.3	6.0
18	5	4.1	0.6	3.3	6.0
19	3	3.7	0.6	3.3	6.0
20	6	4.6	0.6	3.3	6.0
21	2	3.6	0.6	3.3	6.0
22	3	3.3	0.6	3.3	6.0
23	1	2.4	0.6	3.3	6.0
24	5	3.4	0.6	3.3	6.0
Average	3.3				
Standard deviation	1.82				
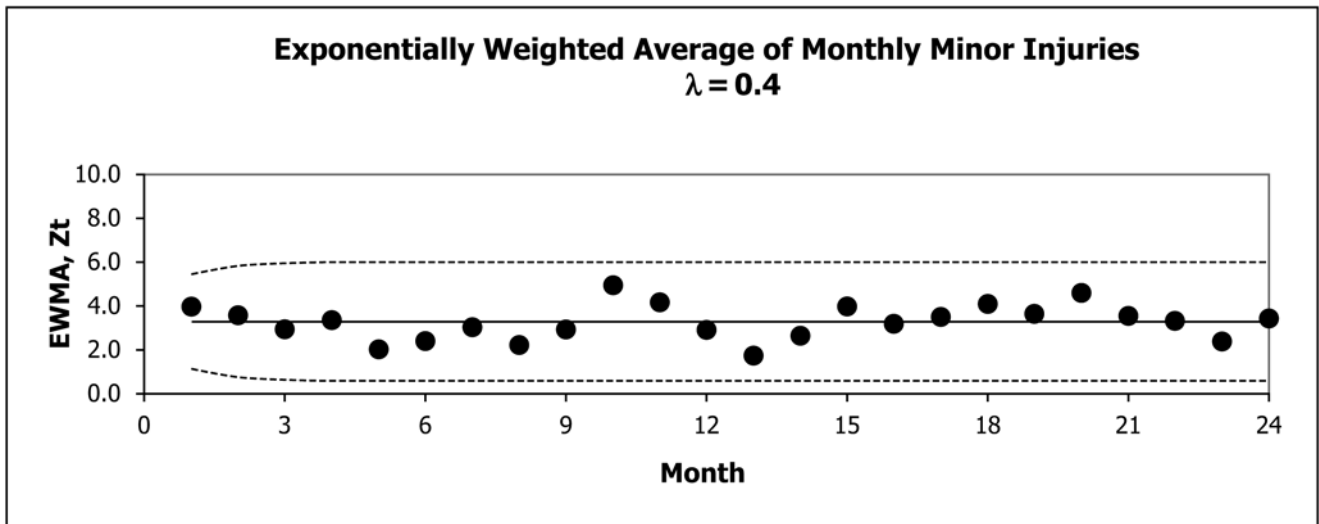
FIG. X1.3 EWMA Chart for Monthly Minor Injuries

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